Data-driven and distributed optimization for addressing complexity in contemporary applications

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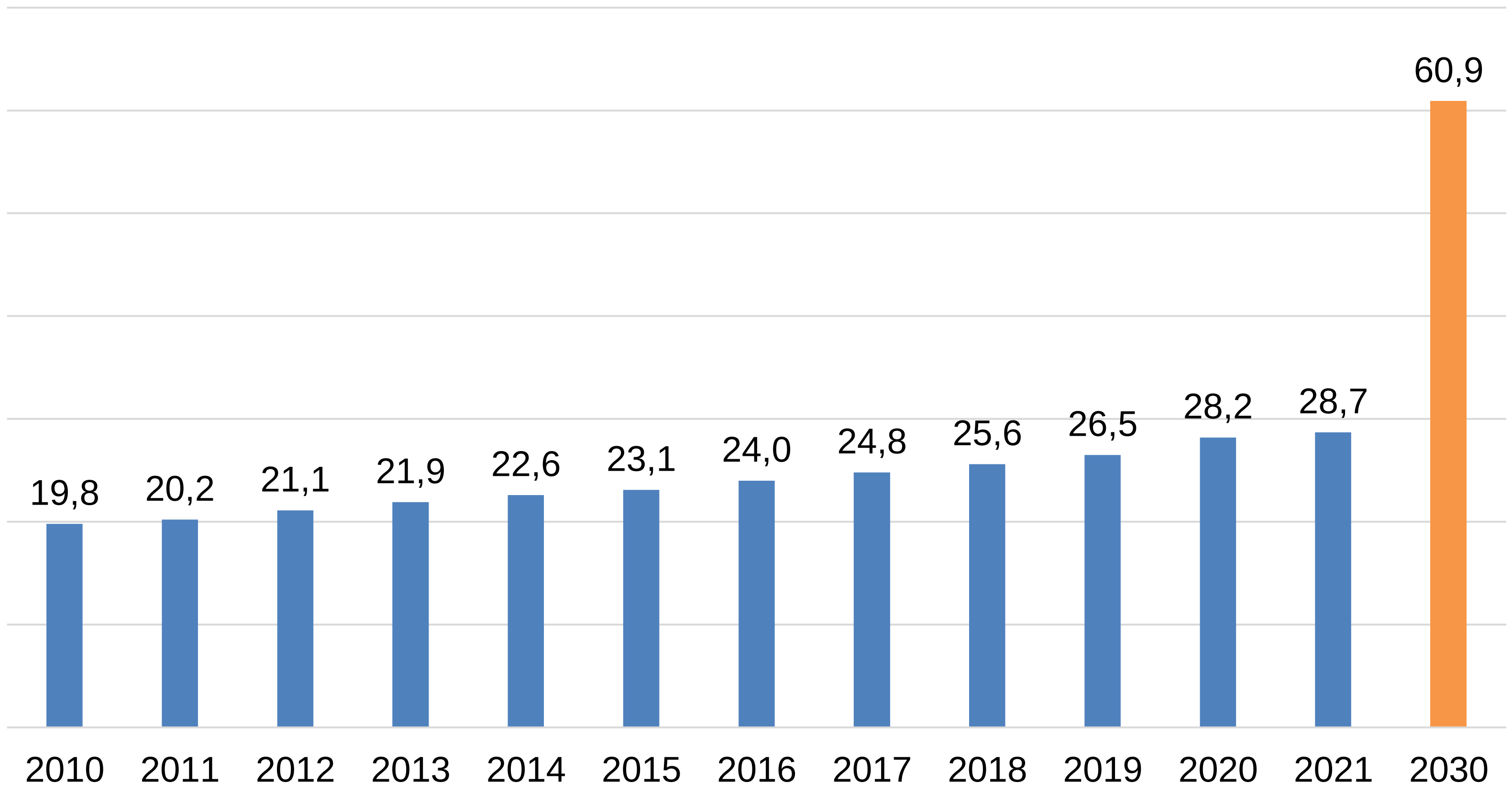
Net zero by 2050

- Global greenhouse gas emissions have to be reduced by 45% by 2030 to reach net zero by 2050
- The energy sector is the source of around three-quarters of greenhouse gas emissions.
- Polluting fossil-fueled power needs to be replaced with renewable energy.

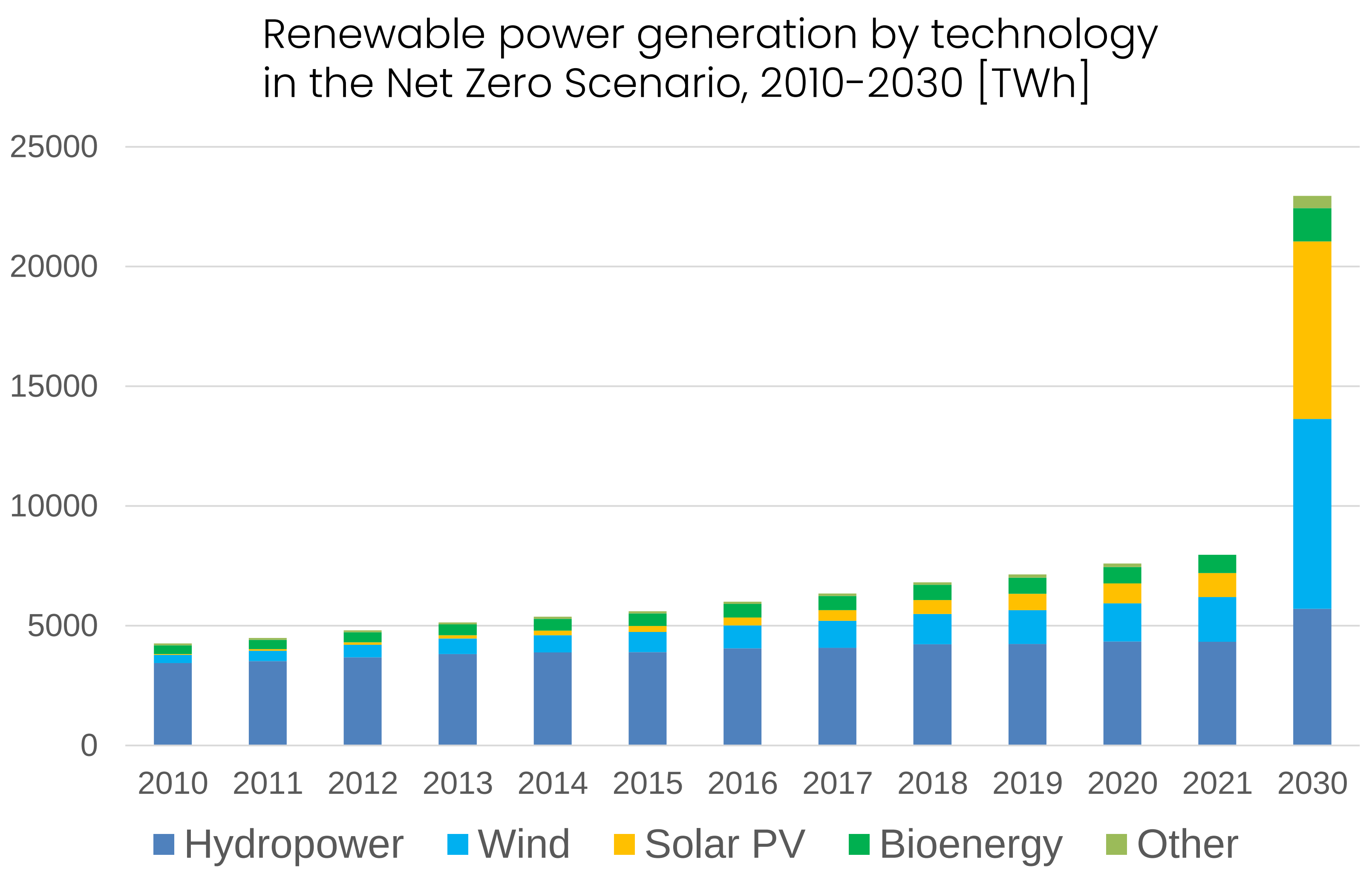
Net zero by 2050

Renewables share of power generation
in the Net Zero Scenario, 2010-2030



From the International Energy Agency report on renewable electricity – Sept 2022

Net zero by 2050



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Integration of solar and wind power in the grid

Widespread increase in distributed energy production from solar and wind generators:

- Reduction of global greenhouse gas emissions
- Energy production close to where it is consumed

Integration of solar and wind power in the grid

Widespread increase in distributed energy production from solar and wind generators:

- ➕ Reduction of global greenhouse gas emissions
- ➕ Energy production close to where it is consumed
- ➖ Risk of destabilization of the grid due to their non-dispatchable and only partly predictable nature

Transition to sustainable energy production

Challenge:

manage the day-to-day differences between supply and demand

Main directions:

- replace coal by other sources like methane that are less polluting but still capable of guaranteeing a plannable power supply in the transitionary phase
- add flexibility via:
  - energy storage systems
  - demand response

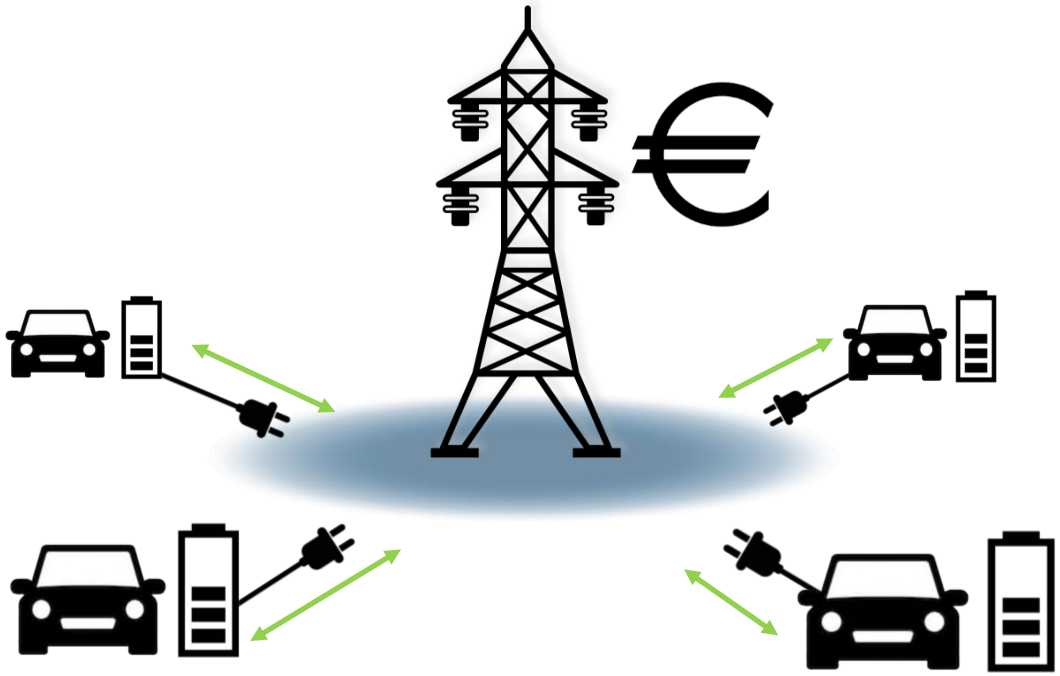
Demand response and end-users involvement

Demand response is the voluntary change of electricity use by end-users.

- implicit demand response
  price signals and tariffs are used to incentivize consumers to shift consumption

Charging of a fleet of electric vehicles

Goal of the fleet aggregator: decide the schedule for each vehicle so as to minimize the charging cost of the whole fleet while satisfying local and global constraints



Each vehicle:

- Final SoC + Battery capacity constraints
- V2G: charge/discharge at some bounded rate

Distribution network:

- Maximum power exchange limit
- **Costs for charging/discharging**

Demand response and end-users involvement

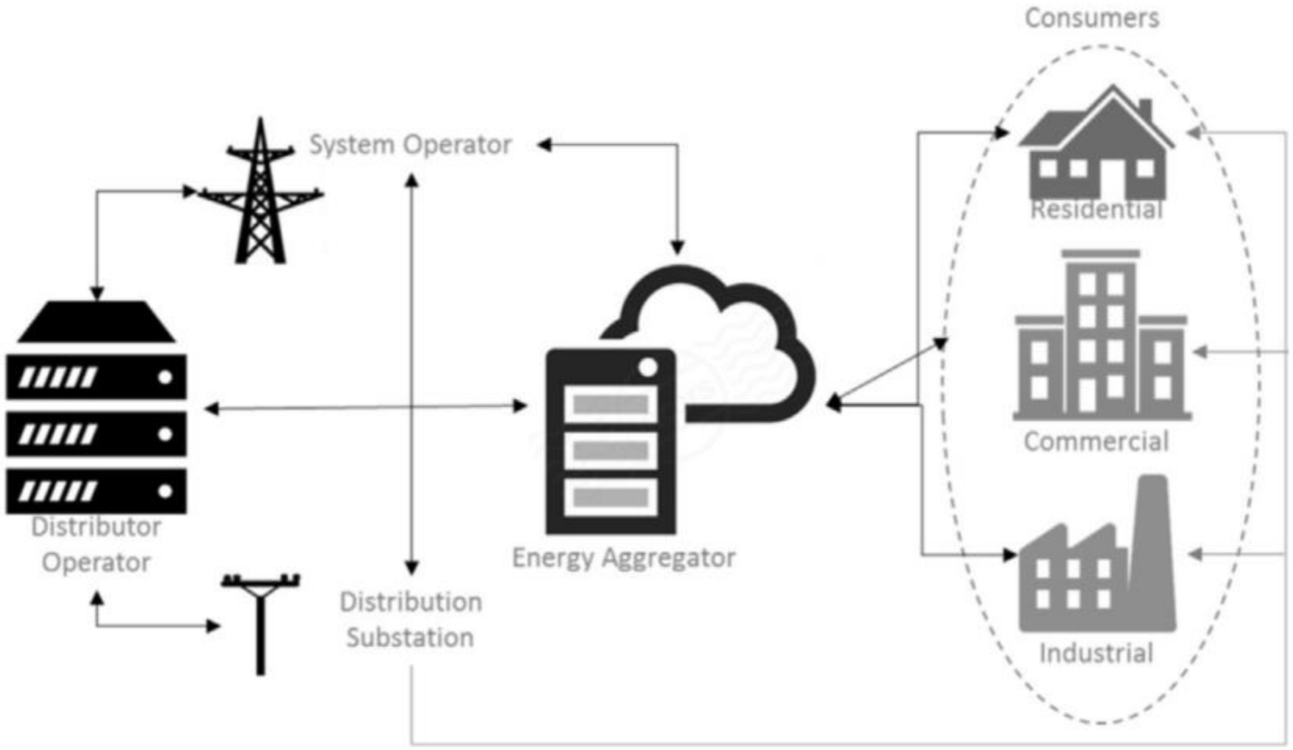
Demand response is the voluntary change of electricity use by end-users.

- implicit demand response
  price signals and tariffs are used to incentivize consumers to shift consumption
- explicit demand response
  flexibility is monetized through direct payments to consumers who shift demand upon request

The Energy Efficiency Directive (2012/27/EU) requires to allow DR participation in the energy market for providing balancing services

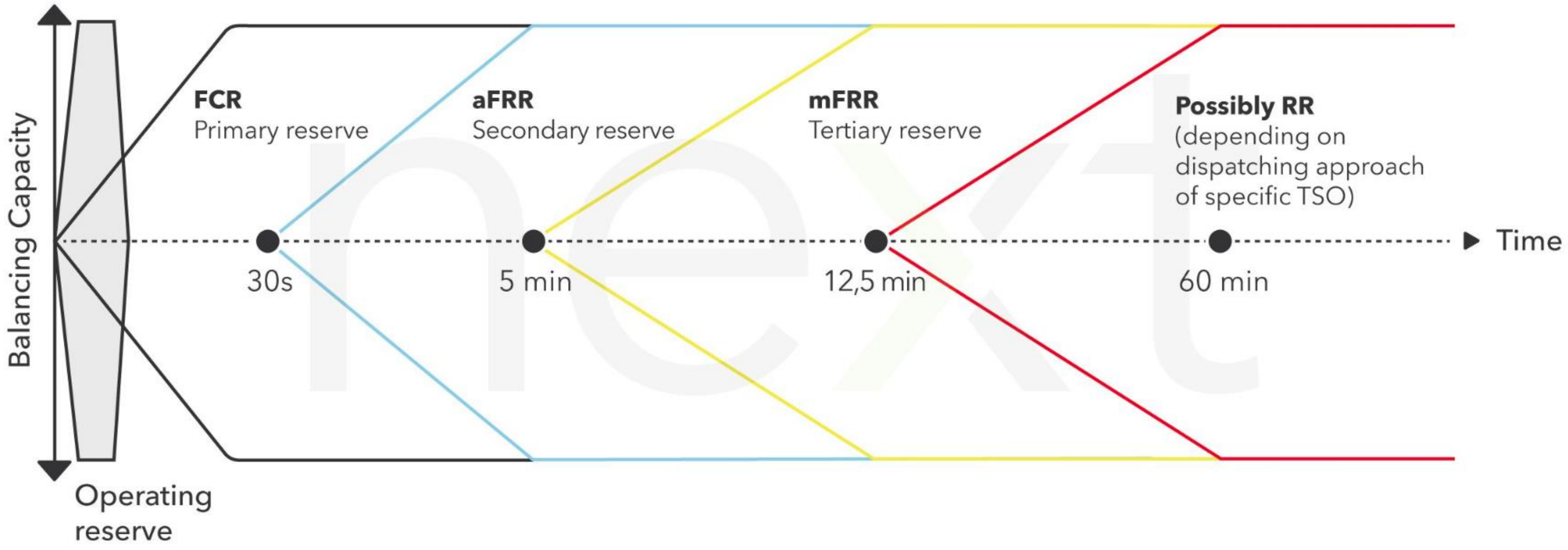
Manual Frequency Restoration Reserve via End-users Aggregation

Balancing Services Providers (BSPs) aggregate end-users and offer manual frequency restoration reserves (mFRR) to the grid operator



Balancing products in the ENTSO-E area

In the European Network of Transmission System Operators for Electricity area, there are four main balancing products:



Manual Frequency Restoration Reserve via End-users Aggregation

1. The grid operator sends a balancing energy request to the BSP
2. The BSP receives the request and optimizes its distribution over the aggregated units asking them to reduce or increase their energy consumption
3. The designated units modulate their consumption/generation level
4. The energy modulation is made available to the grid operator

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Addressed problem

Optimally operating
complex systems

Addressed problem

Optimally operating complex systems

Size

Addressed problem

Optimally operating complex systems

Size

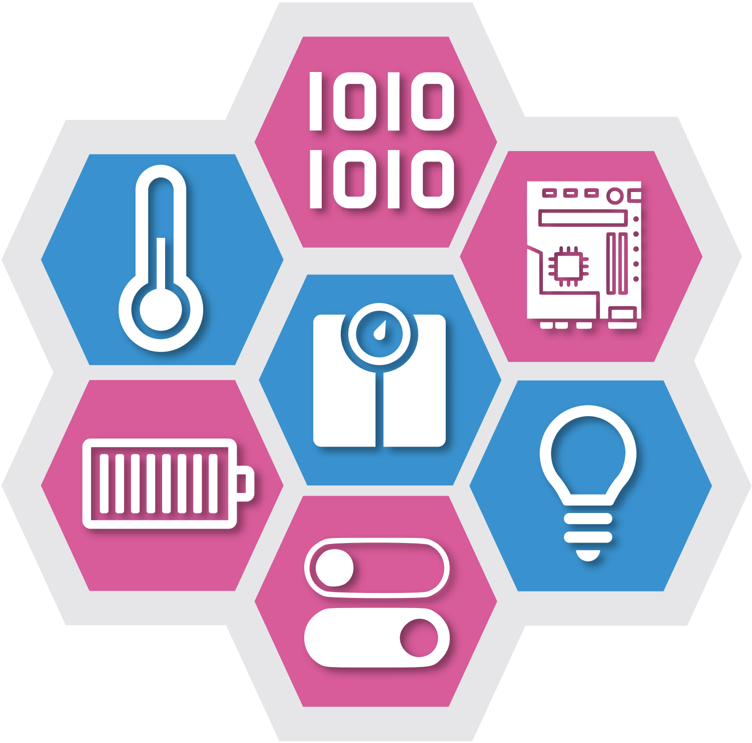
Heterogeneity

Addressed problem

Optimally operating complex systems

- **Size**
- **Heterogeneity**
- **Uncertainty**

Mixed Logical Dynamical system



Modelling framework

Mixed Logical Dynamical system

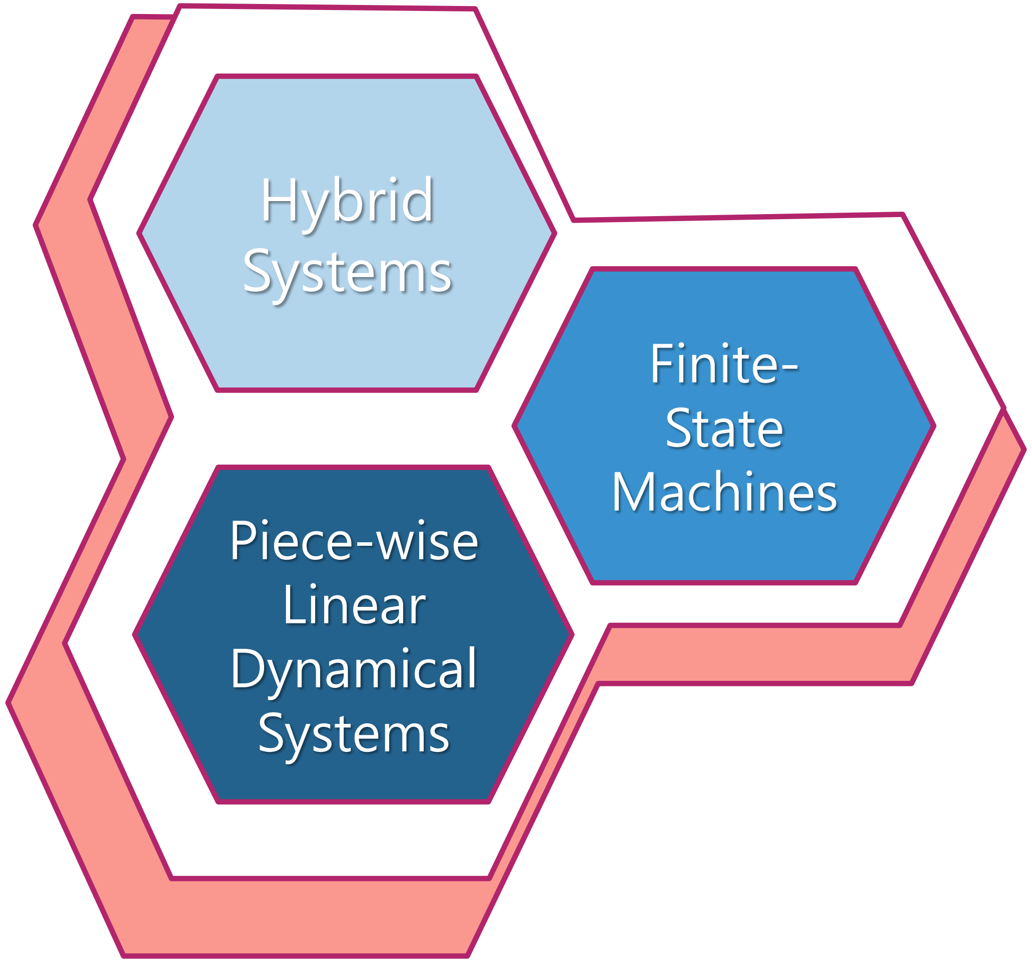
$$s(t+1) = A_t s(t) + B_{1t} u(t) + B_{2t} \eta(t) + B_{3t} z(t)$$

$$E_{2t} \eta(t) + E_{3t} z(t) \leq E_{1t} u(t) + E_{4t} s(t) + E_{5t}$$

$s = \begin{bmatrix} s_c \\ s_l \end{bmatrix}$, $s_c \in \mathbb{R}^{n_c}$, $s_l \in \{0,1\}^{n_l}$ **State**

$u = \begin{bmatrix} u_c \\ u_l \end{bmatrix}$, $u_c \in \mathbb{R}^{n_c}$, $u_l \in \{0,1\}^{n_l}$ **Input**

$\eta \in \{0,1\}^{r_l}$, $z \in \mathbb{R}^{r_c}$ **Auxiliary Variables**



Bemporad Alberto e Morari Manfred, **"Control of systems integrating logic, dynamics, and constraints,"** *Automatica 35 (1999): 407-427.*

Optimal operation of MLD

Mixed Logical Dynamical system

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Mixed Integer Linear Program

$$\min_x \quad c^\top x$$

$$s.t \quad Ax \leq b$$

$$x \in \mathbb{R}^{n_c} \times \mathbb{Z}^{n_d}$$

Difficult to handle

Optimization framework

Decomposition strategy

Efficient decentralized resolution schemes

Data-based approach to deal with uncertainty

Optimization framework

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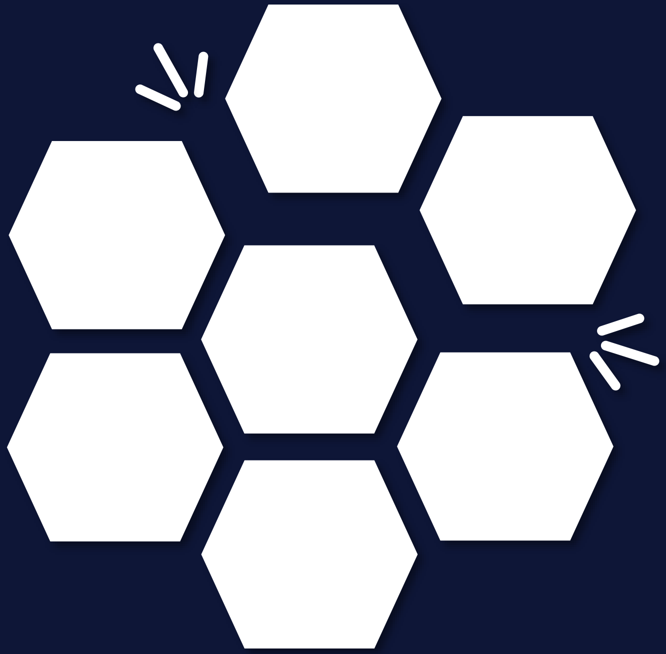

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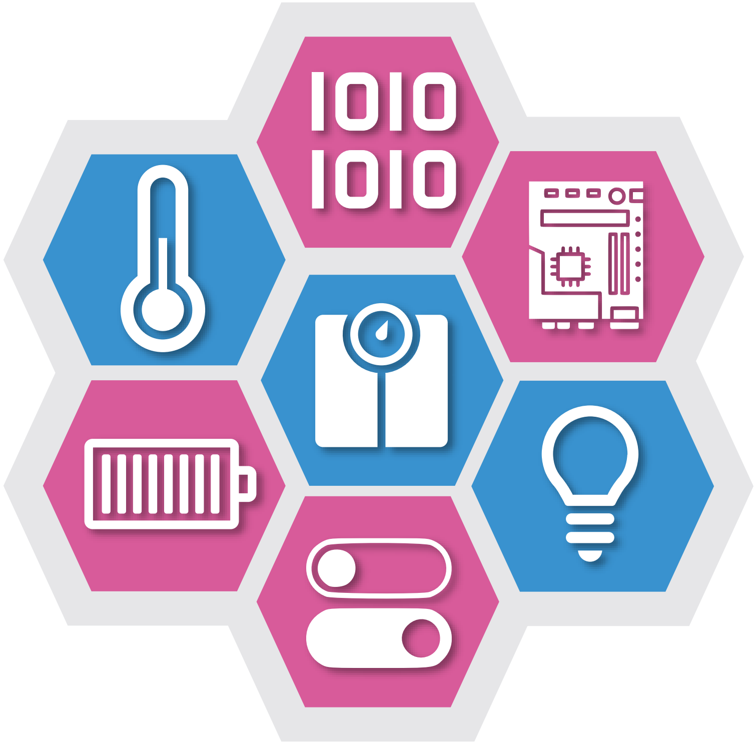

Mixed-Integer Linear Program

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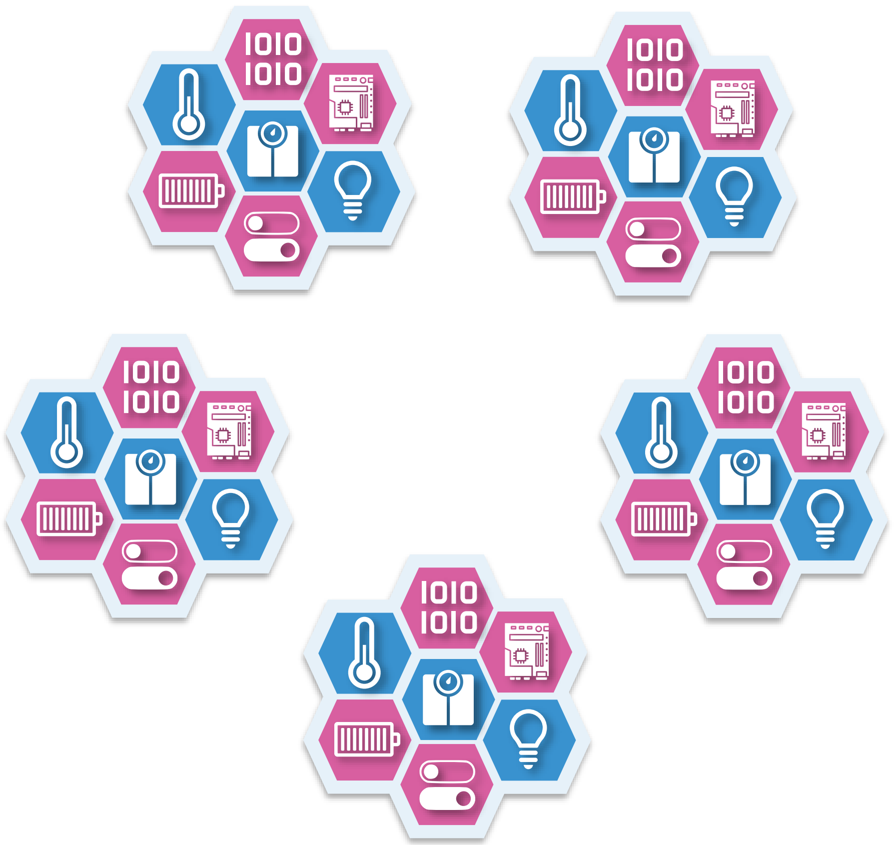
Decomposition strategies

Mixed Logical Dynamical system



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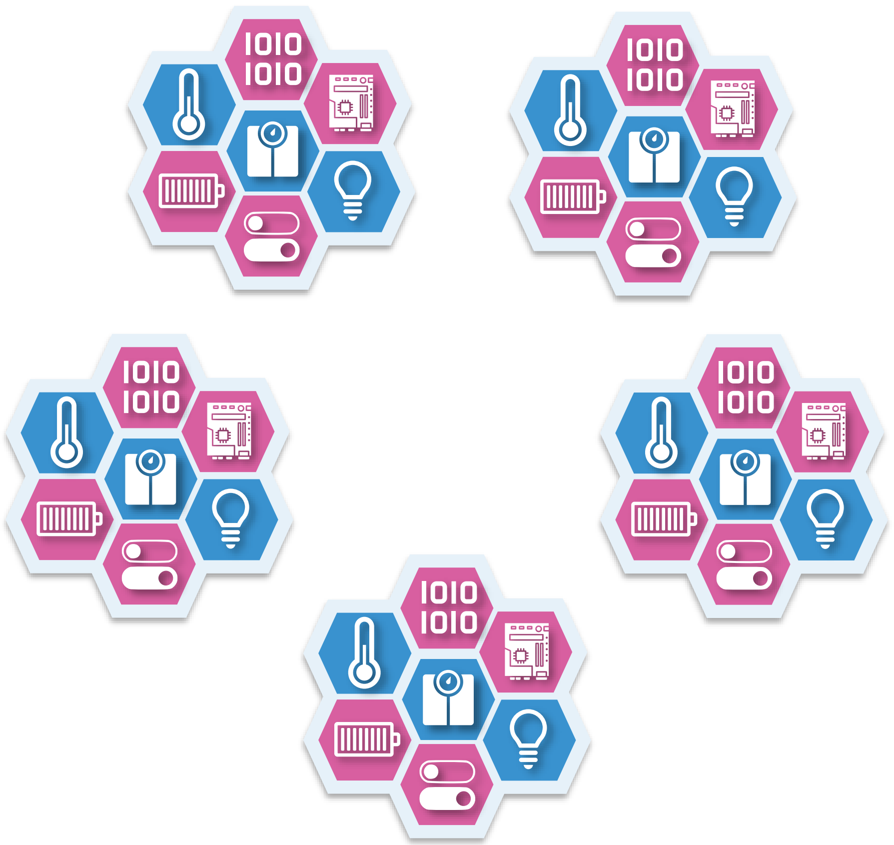

Constraint-coupled multi-agent MILP

$$
\begin{aligned}
\min_{x_1,\dots,x_m} \quad & \sum_{i=1}^{m} \boxed{c_i^\top x_i} \\
\text{s.t.} \quad & D_i x_i \le d_i \quad \forall i \\
& \sum_{i=1}^{m} E_i x_i \le f \\
& x_i \in \mathbb{R}^{n_{c_i}} \times \mathbb{Z}^{n_{d_i}} \quad \forall i
\end{aligned}
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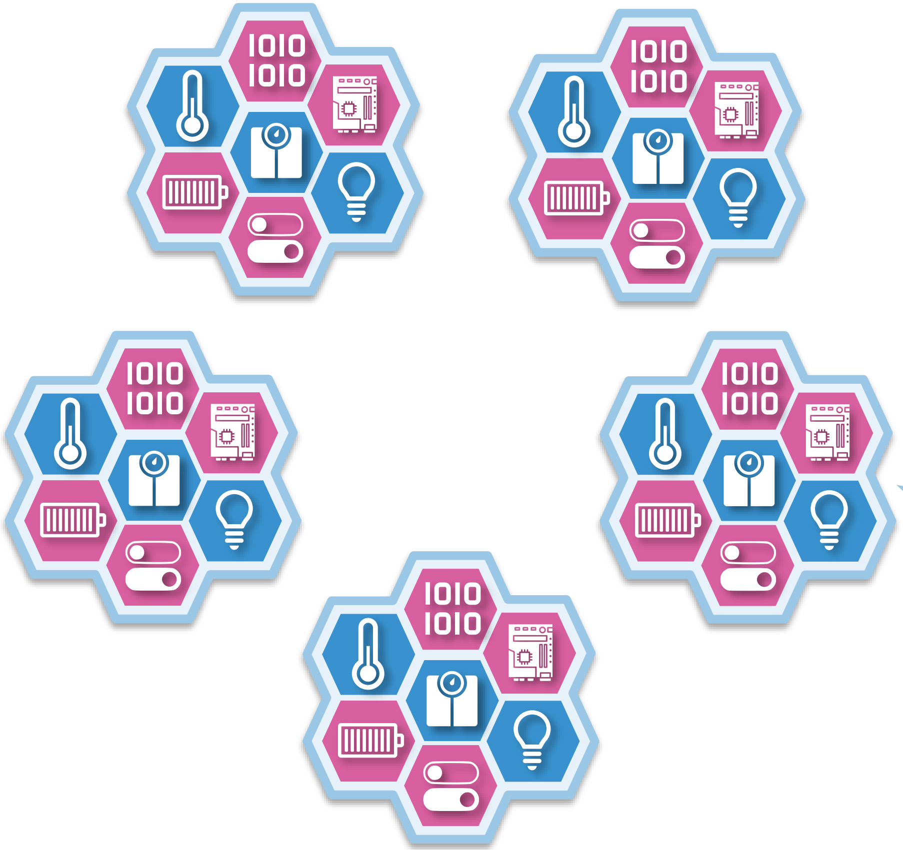
Local Cost

Decomposition strategies

Mixed Logical Dynamical system



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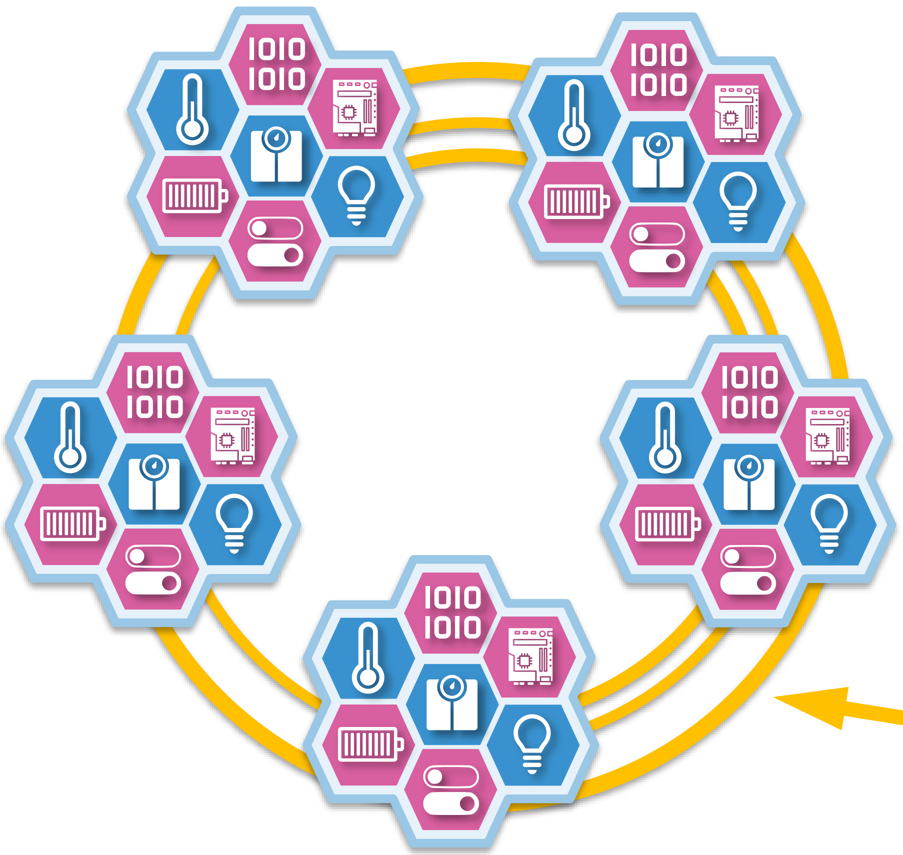
Local Constraints

Decomposition strategies

Mixed Logical Dynamical system



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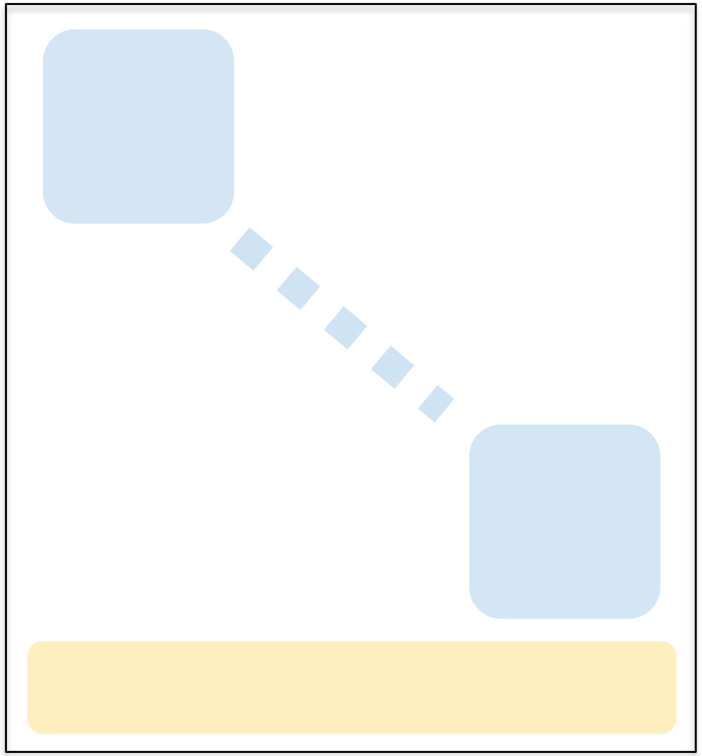
Coupling Constraints

Decomposition strategies

Singly-Bordered block-diagonal matrix



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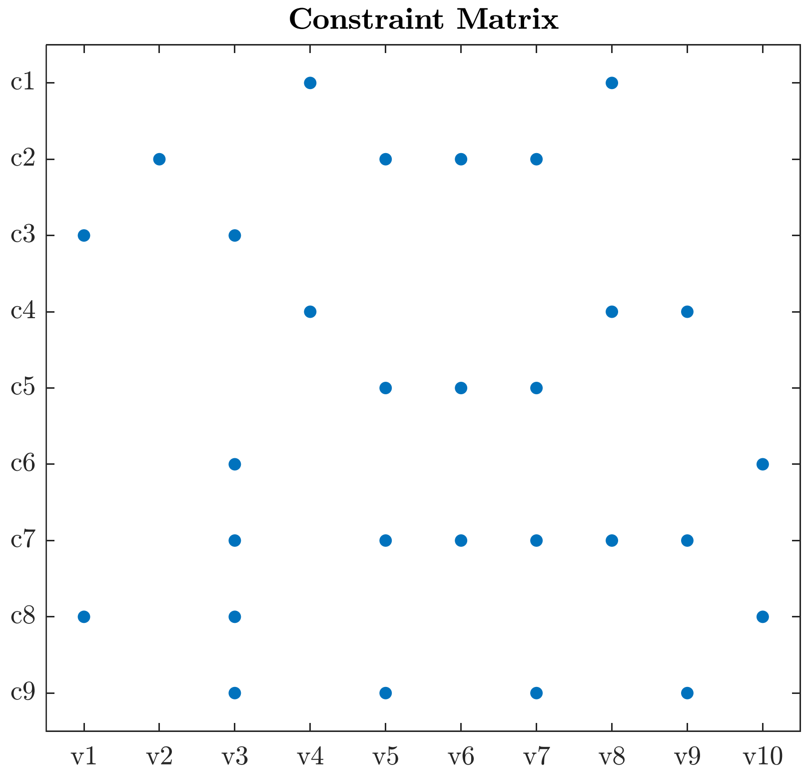
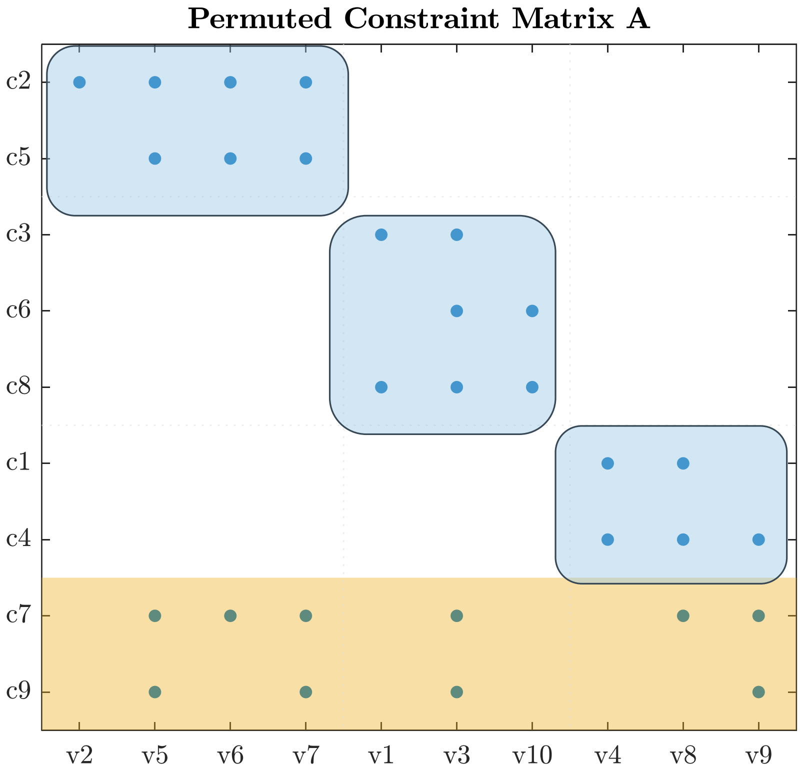
Decomposition strategies





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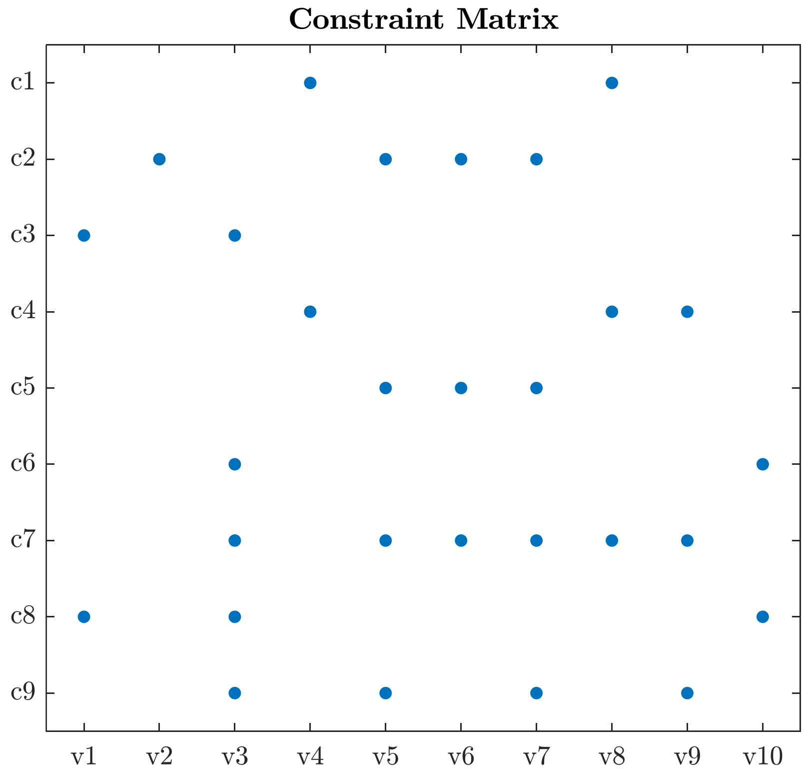

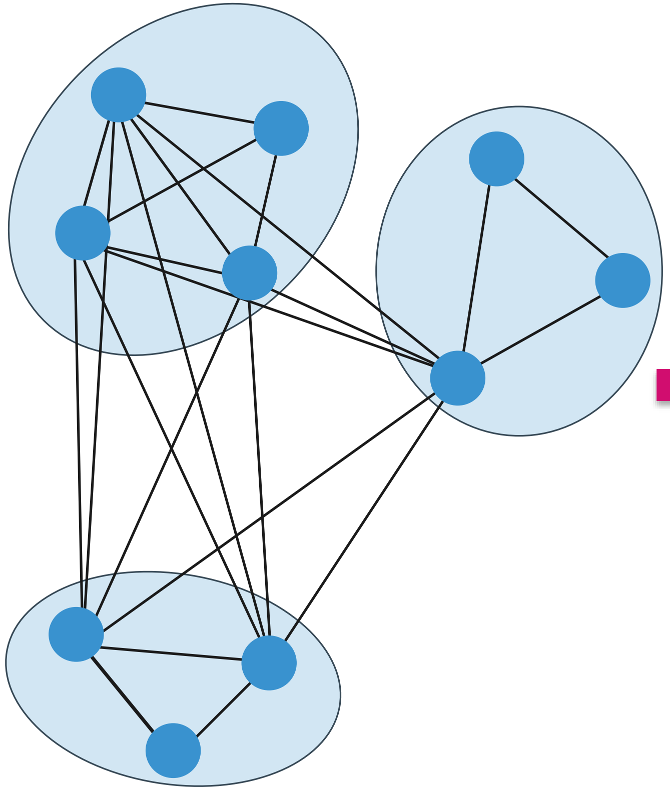

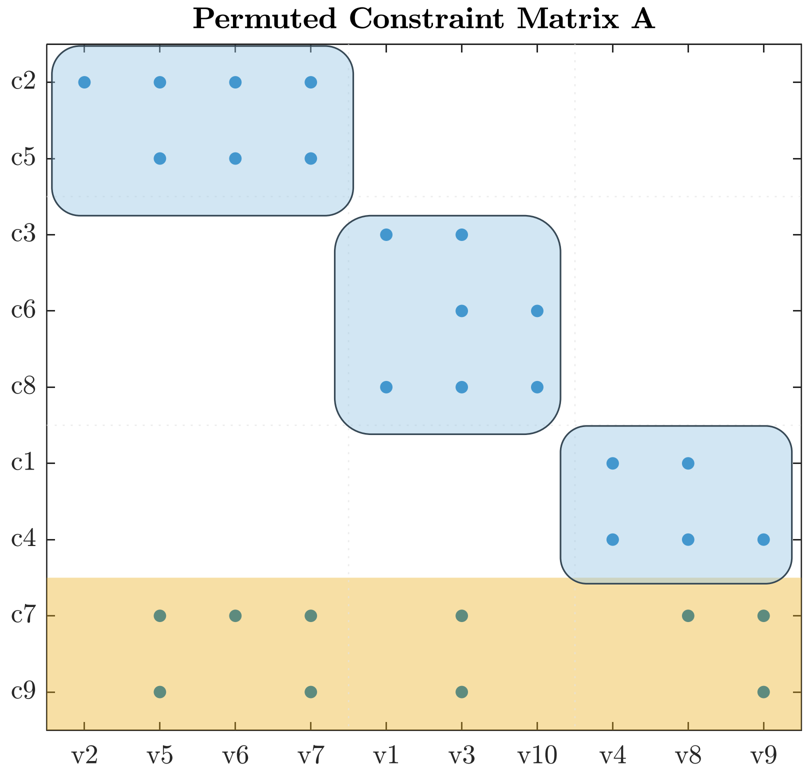

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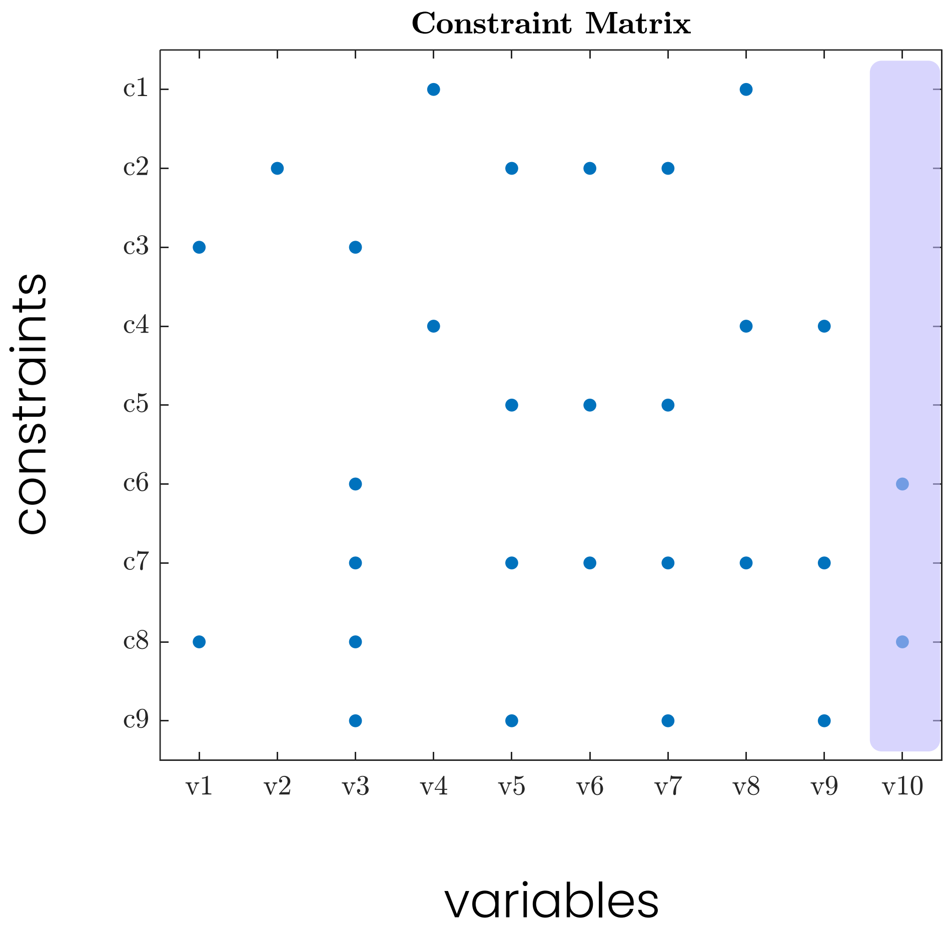

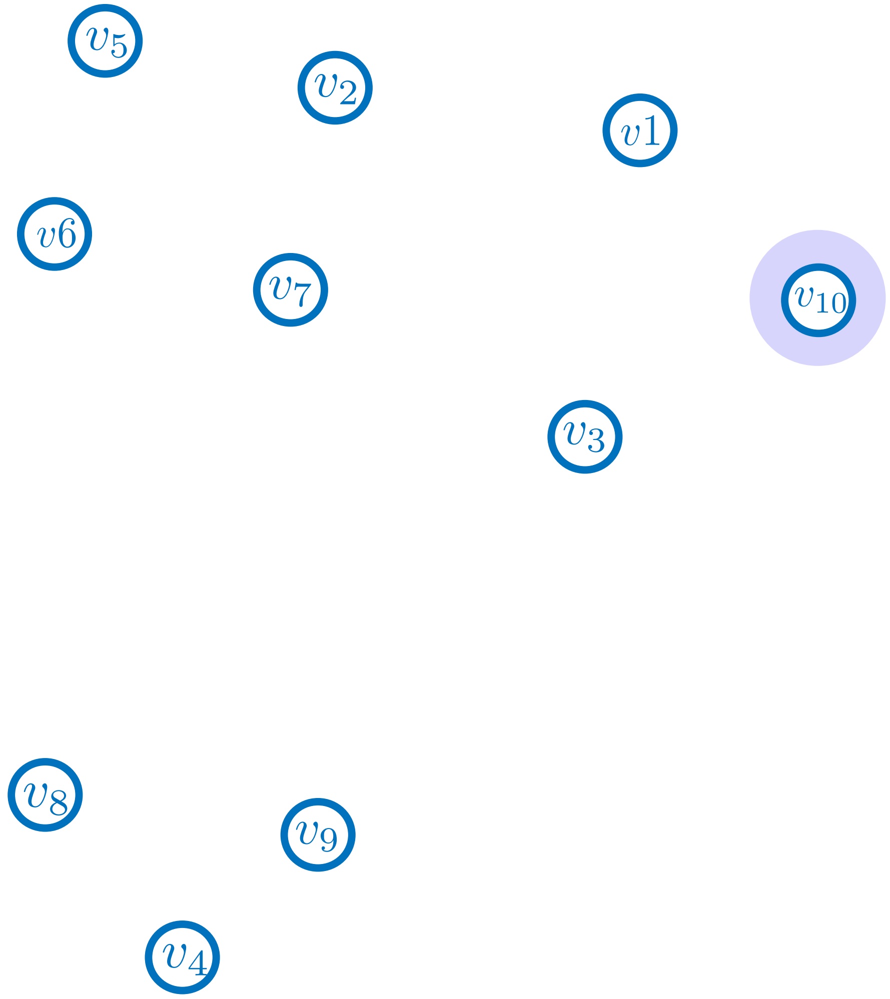

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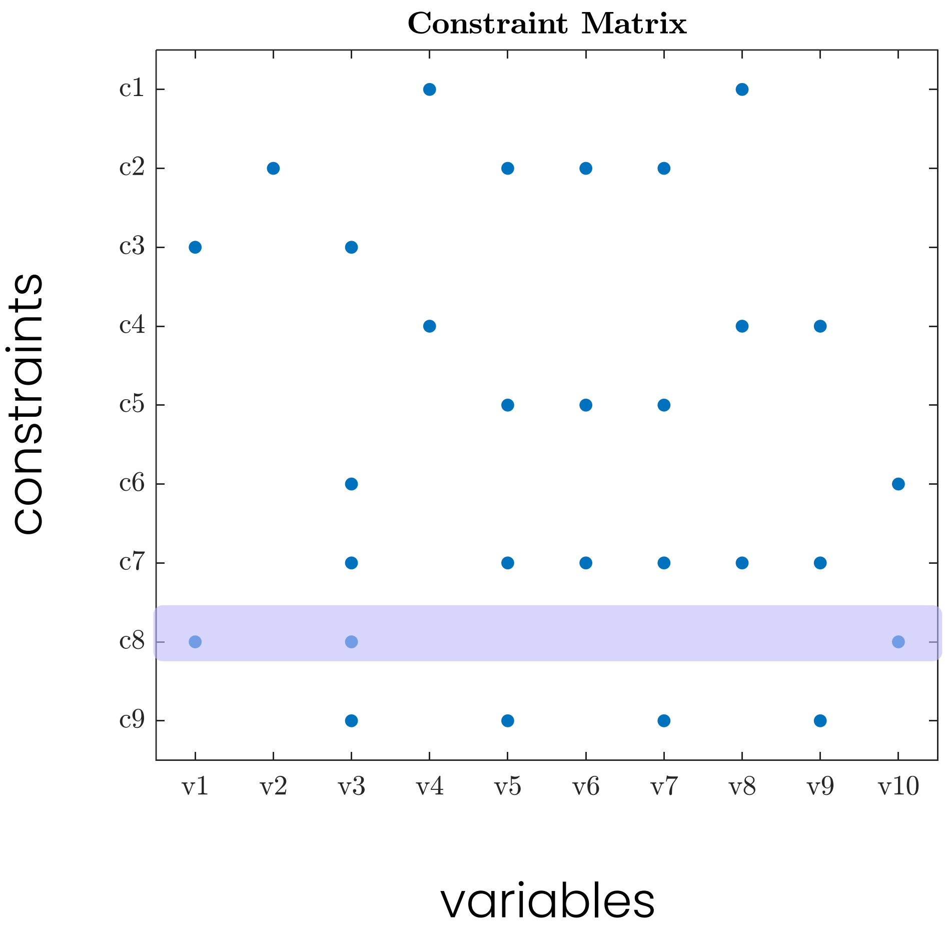

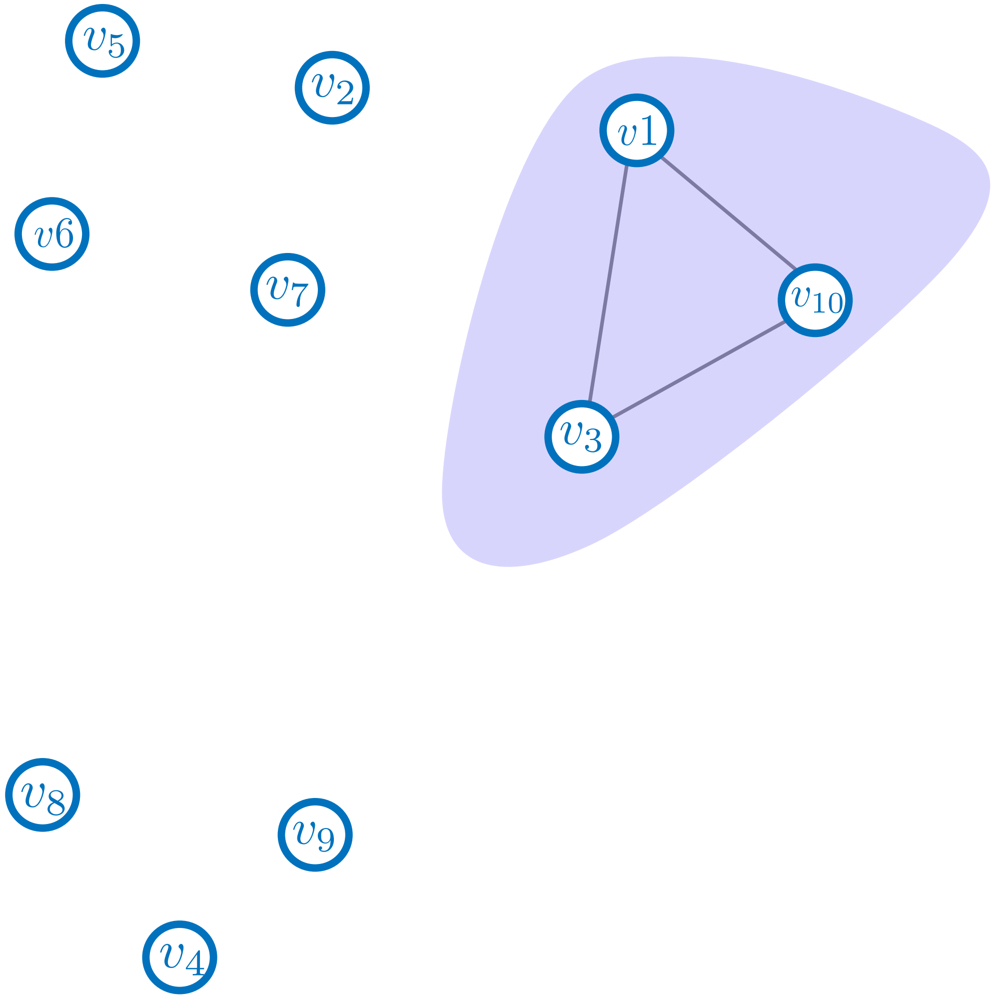

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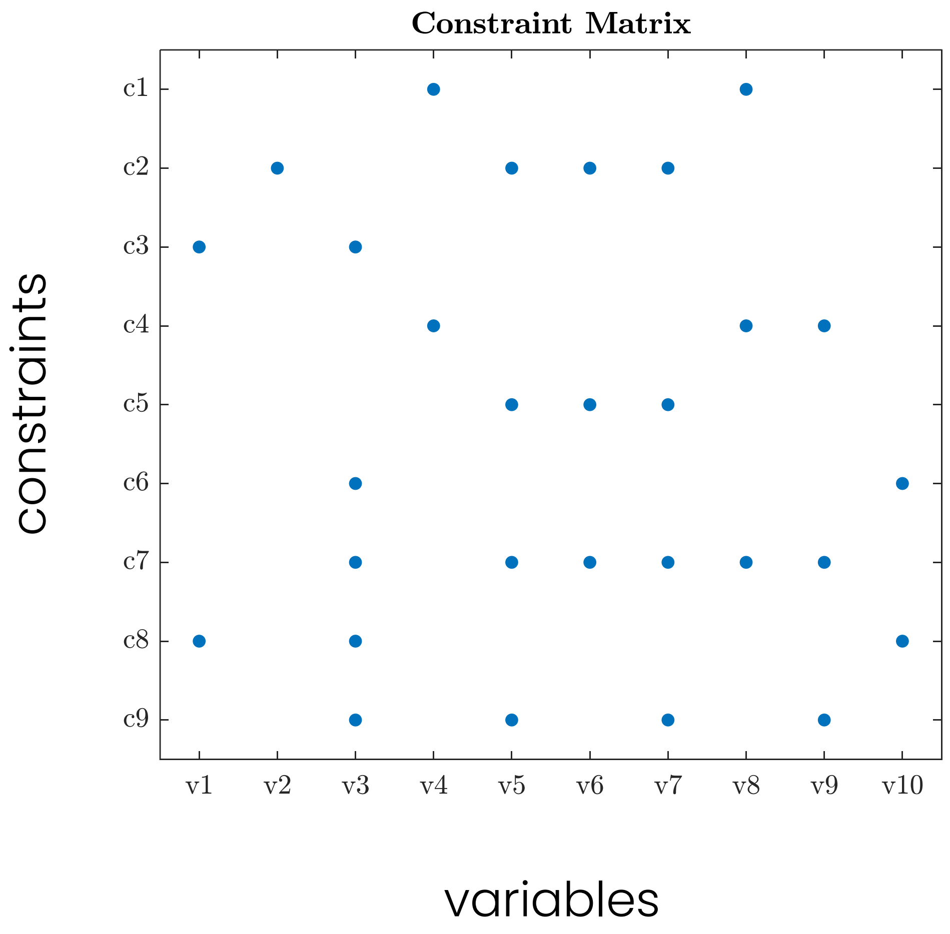

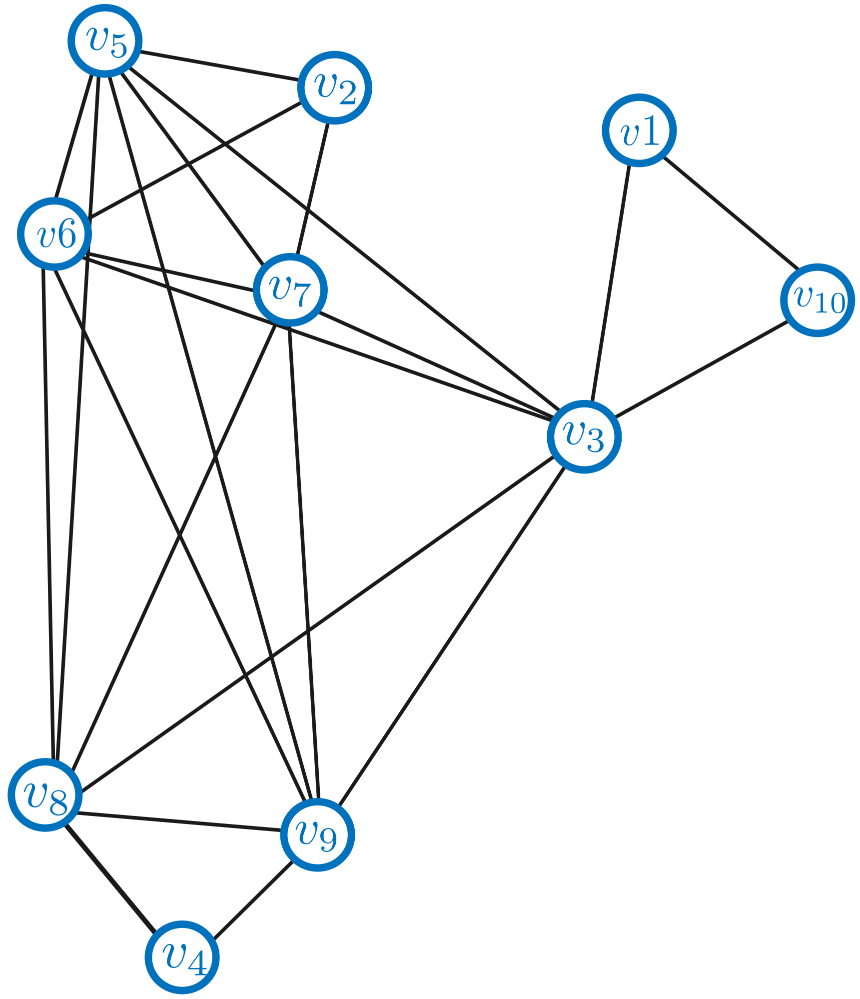

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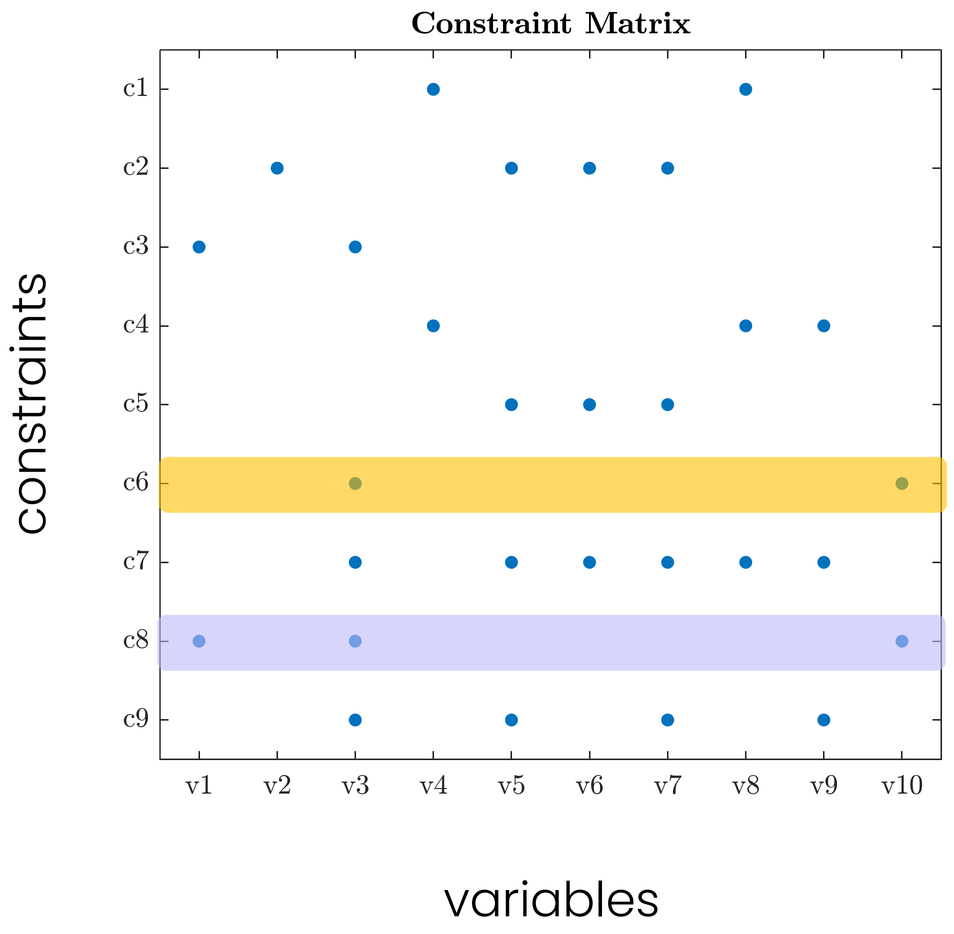
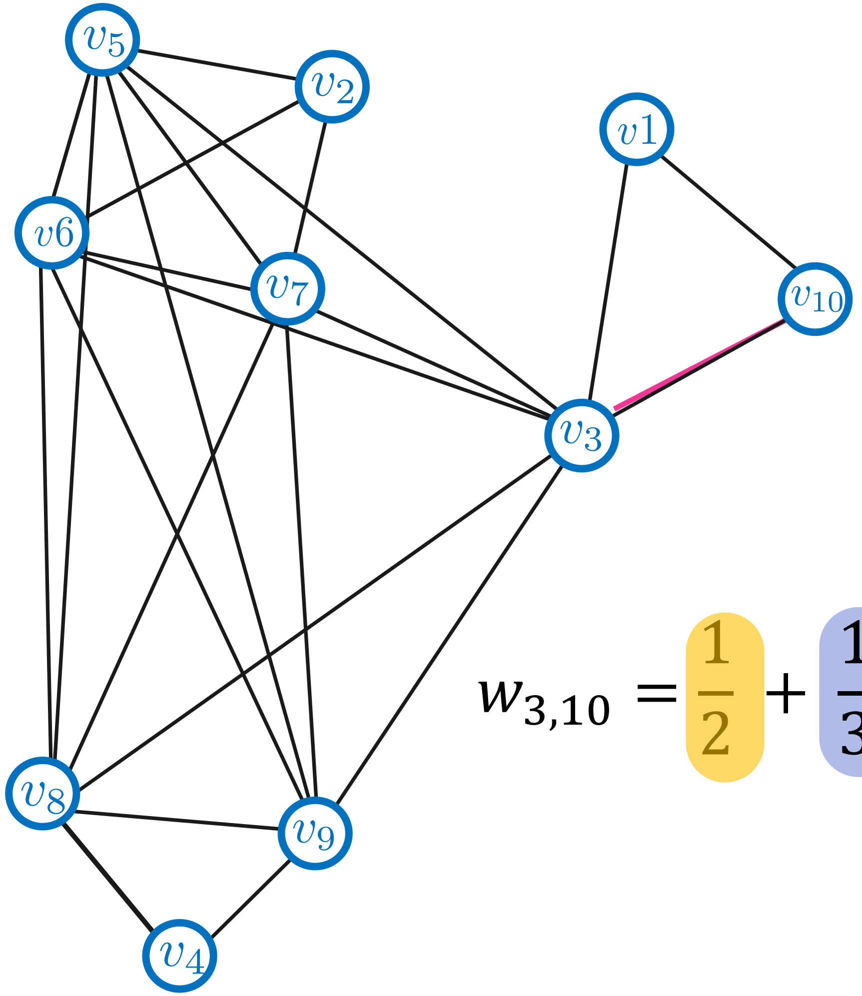
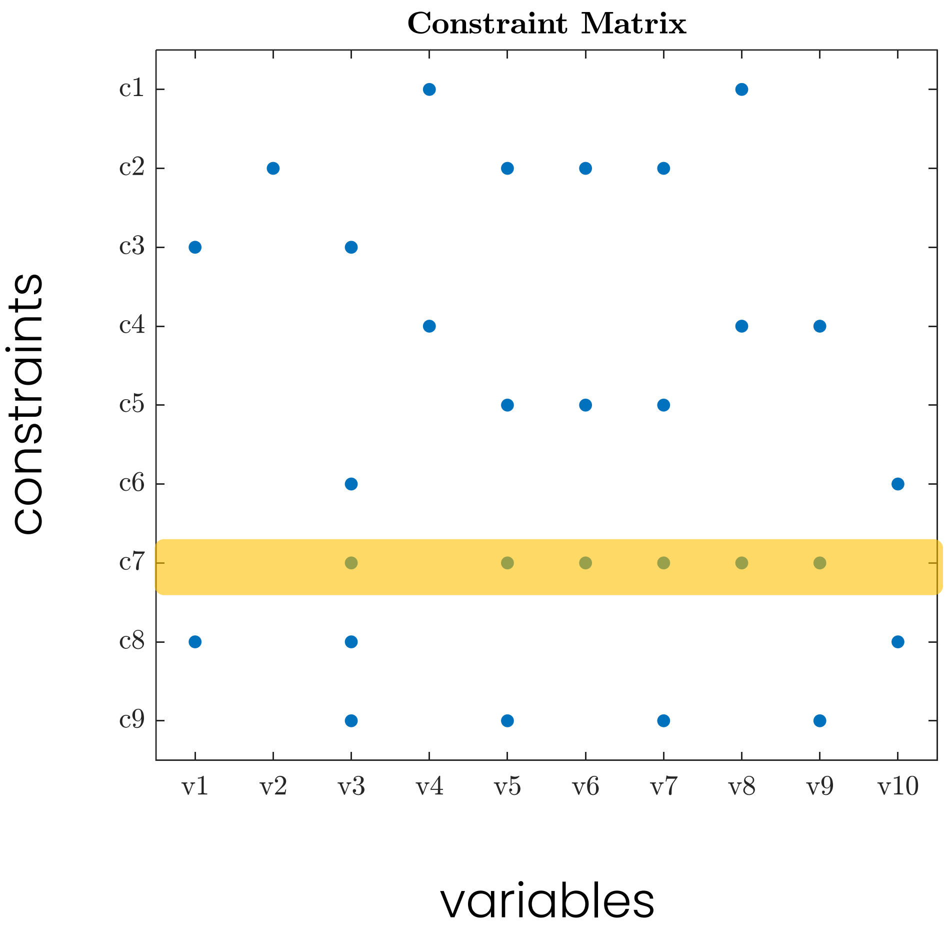
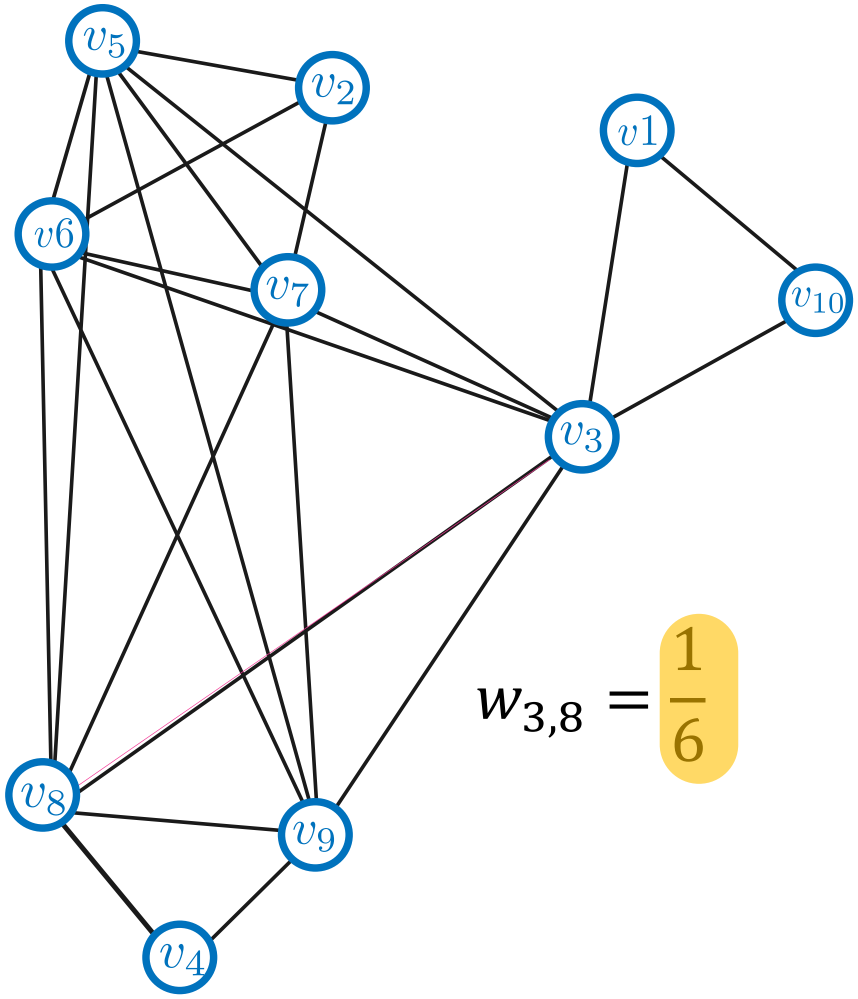




$$w_{3,10} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

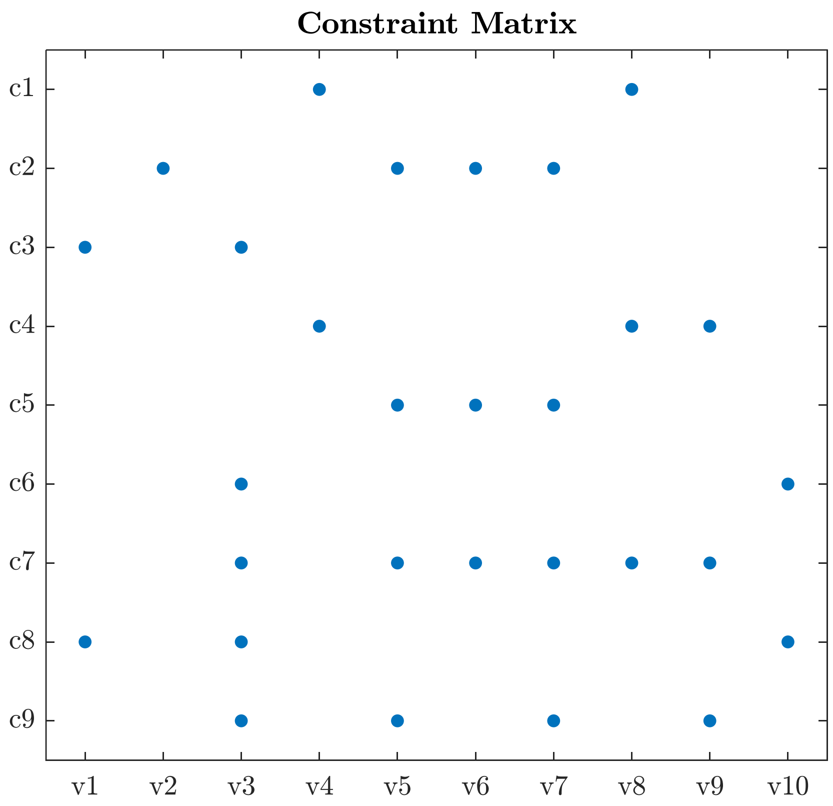
Decomposition strategies

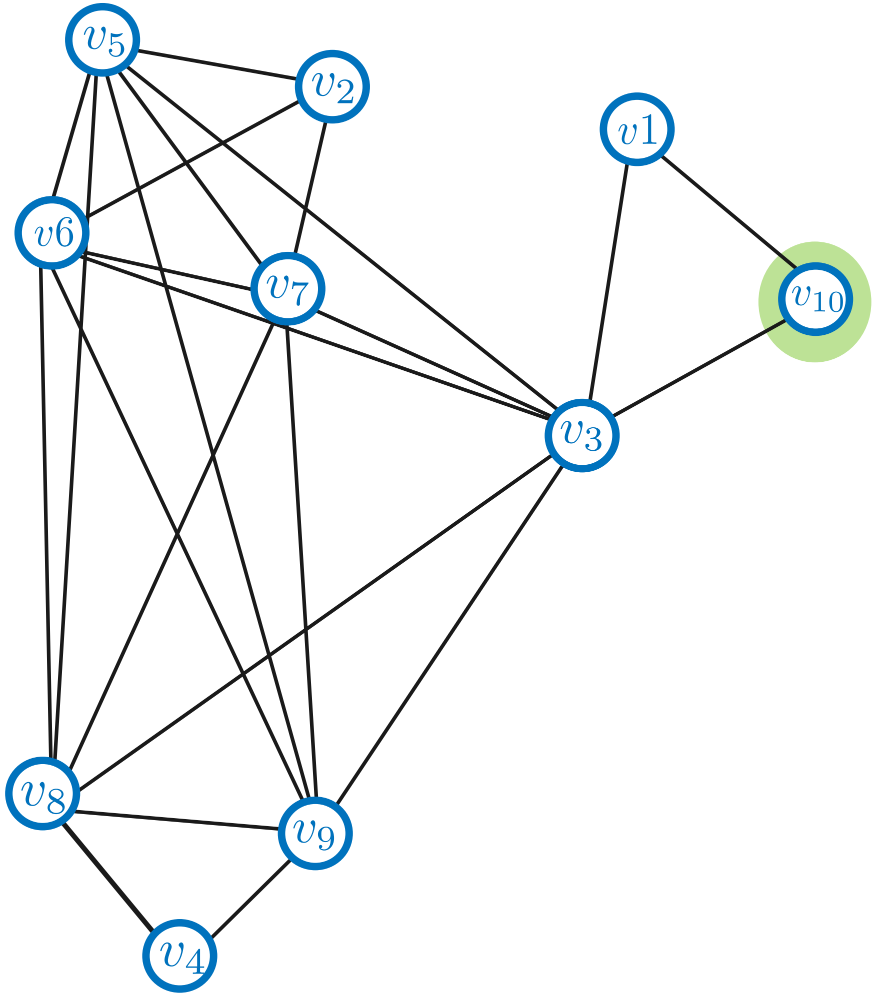




$$w_{3,8} = \frac{1}{6}$$

Decomposition strategies





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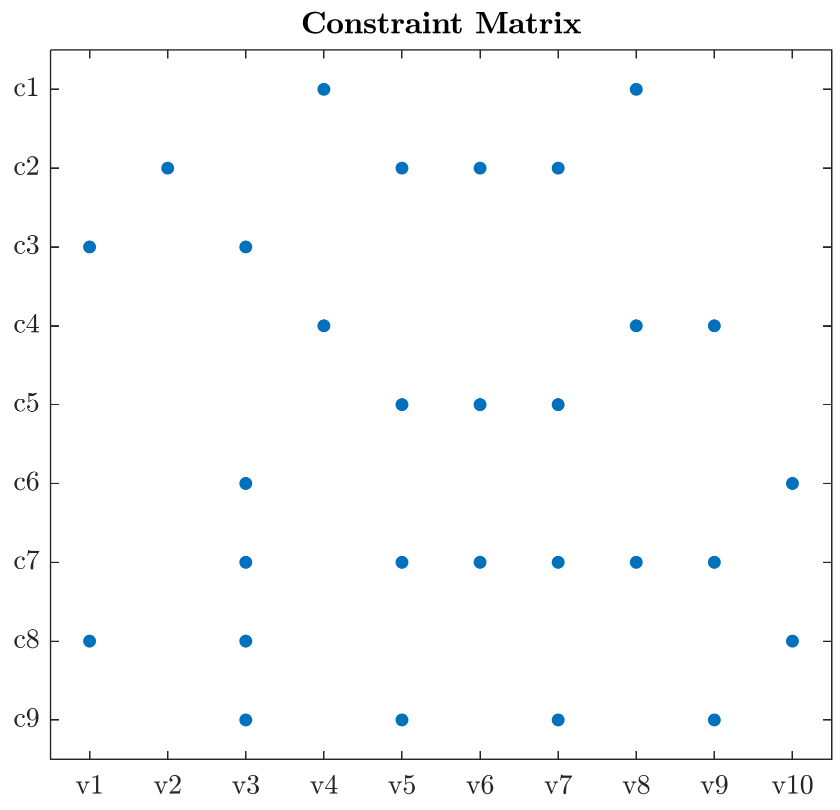

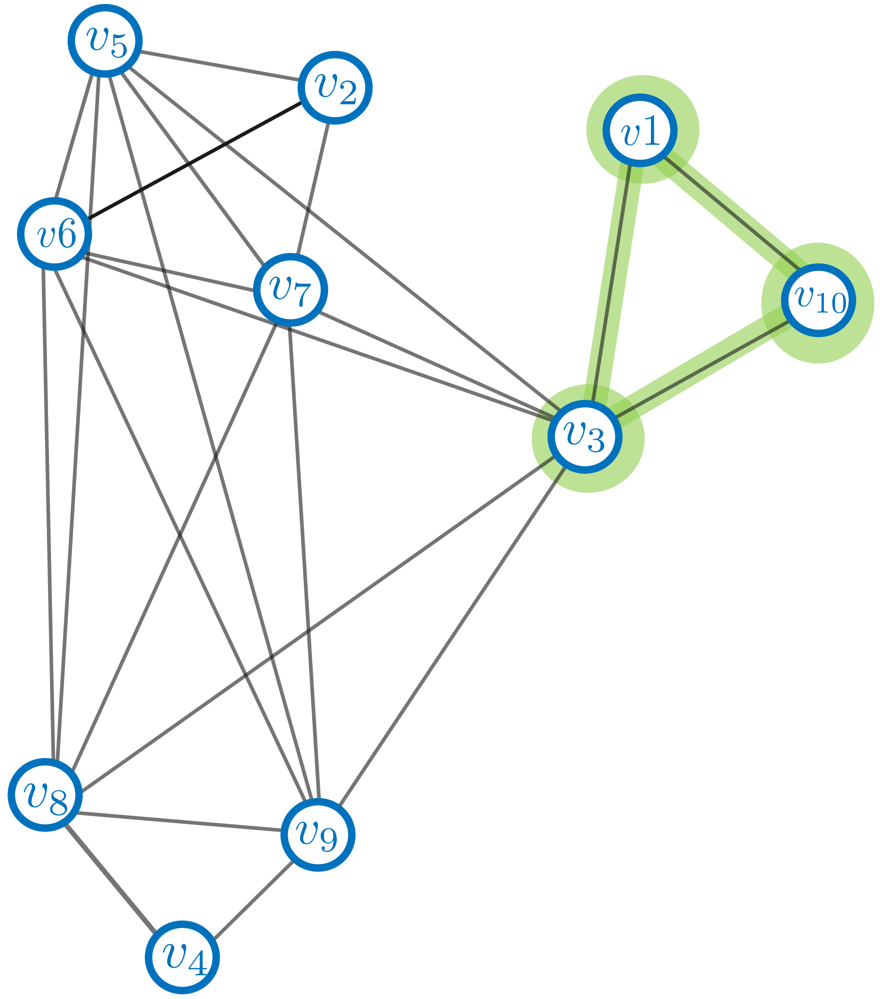

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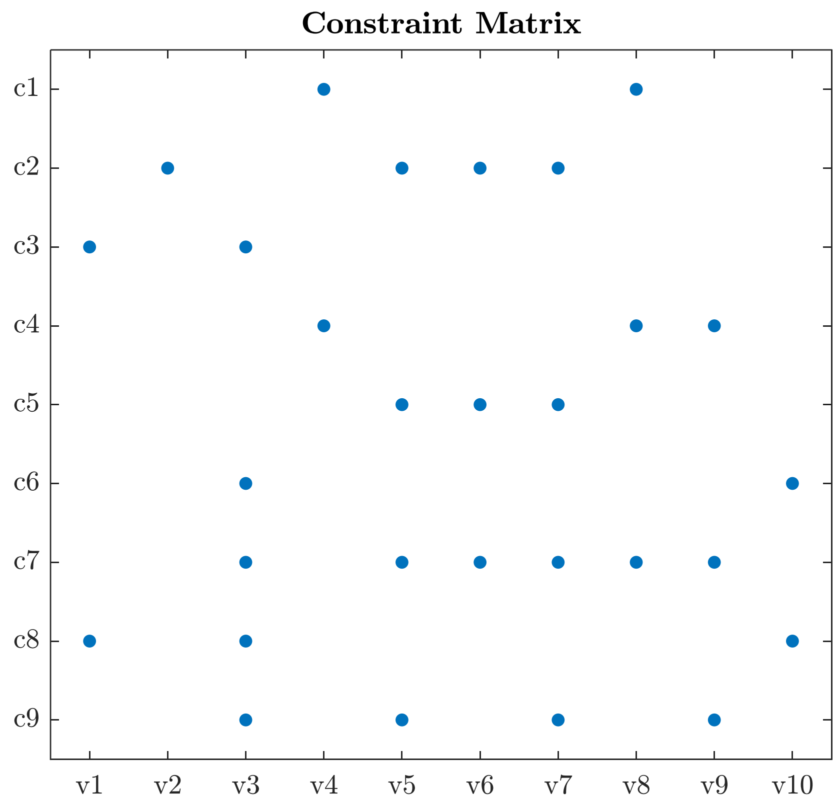

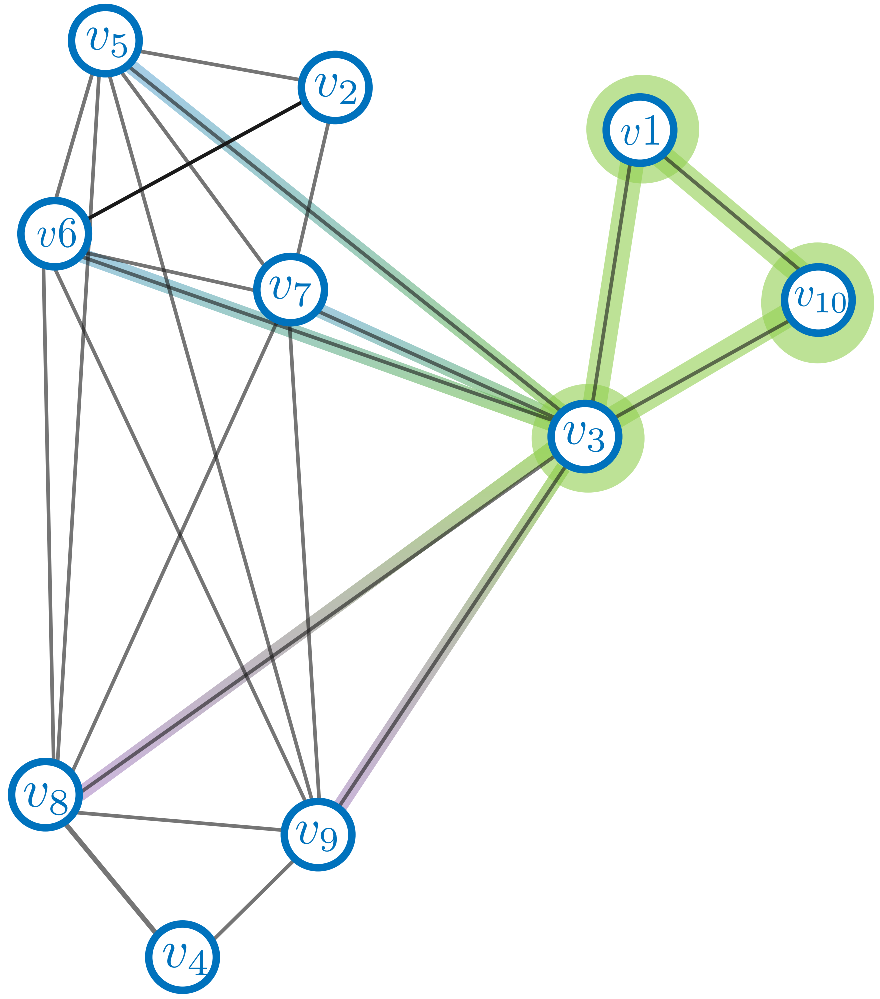

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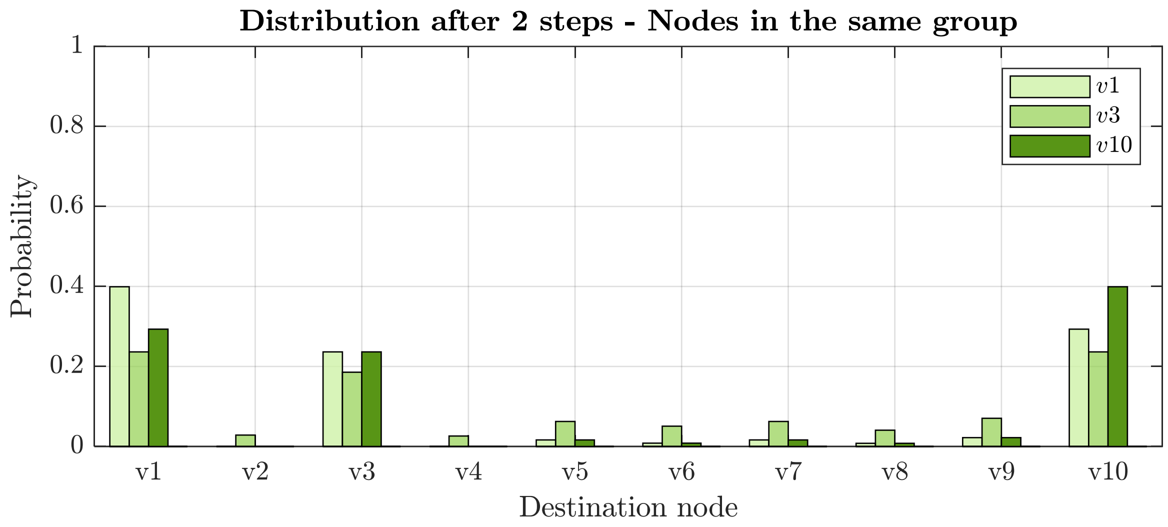

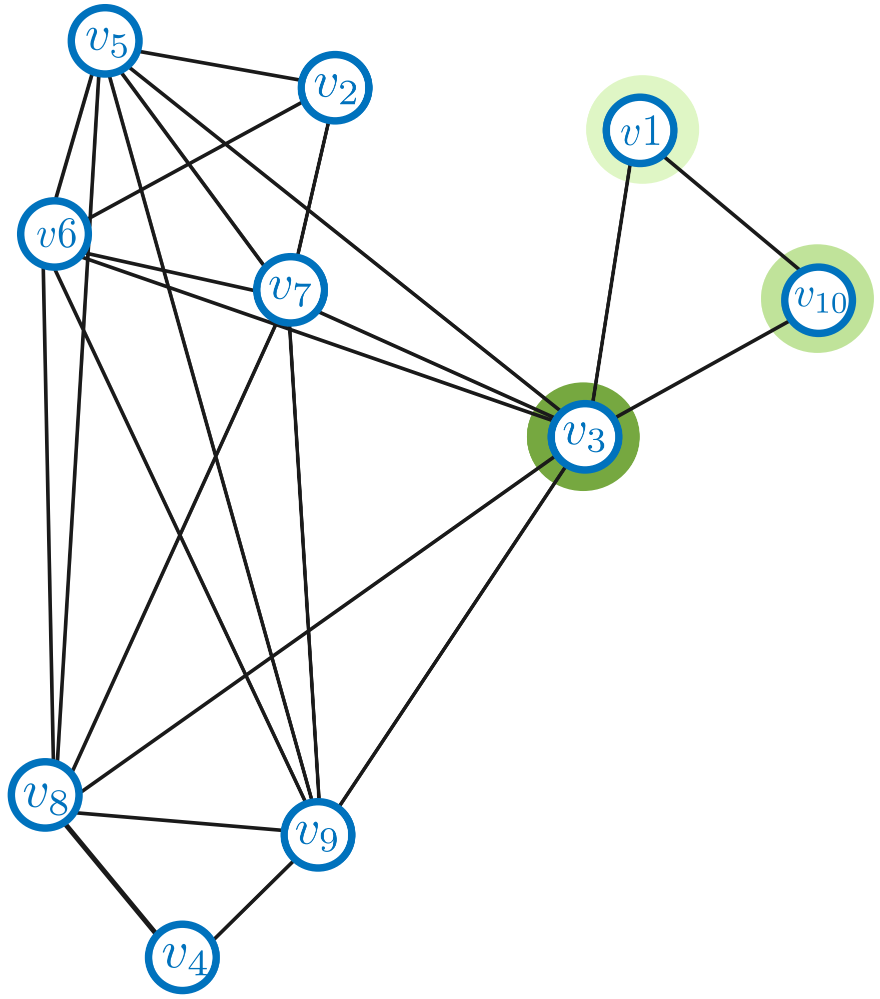

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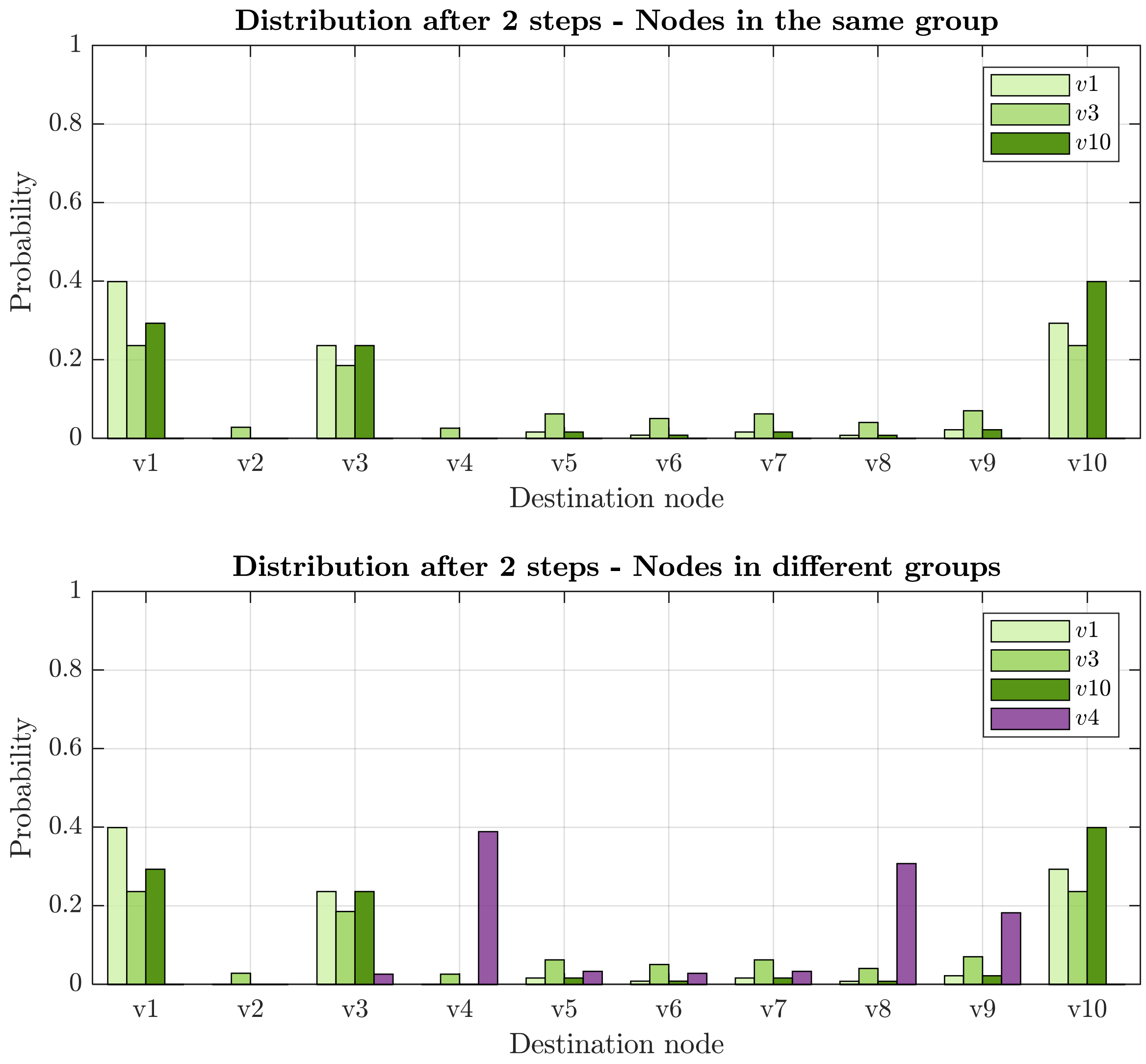
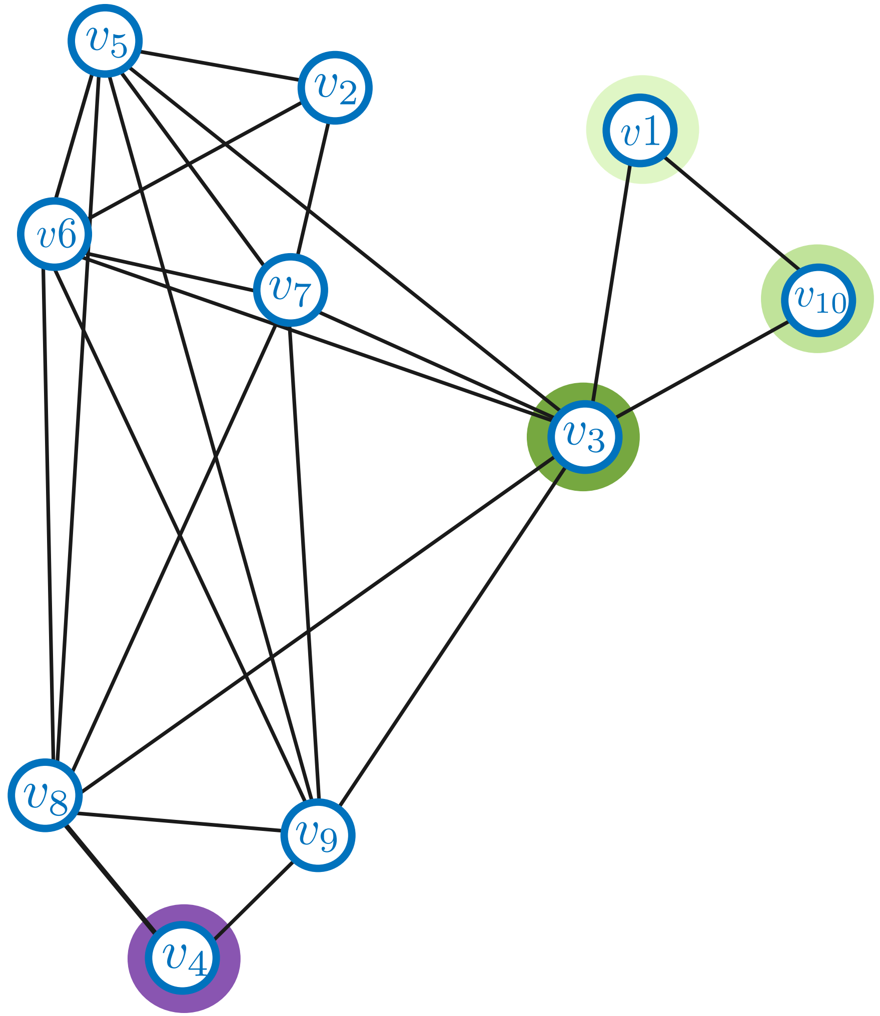




Decomposition strategies

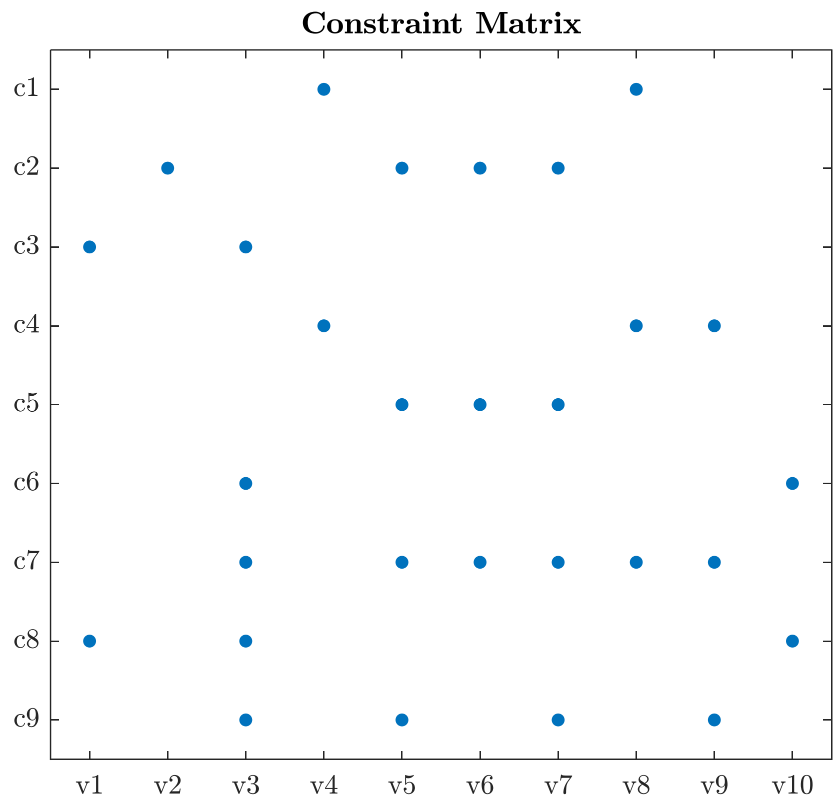
Markov chain representation of the constraint matrix

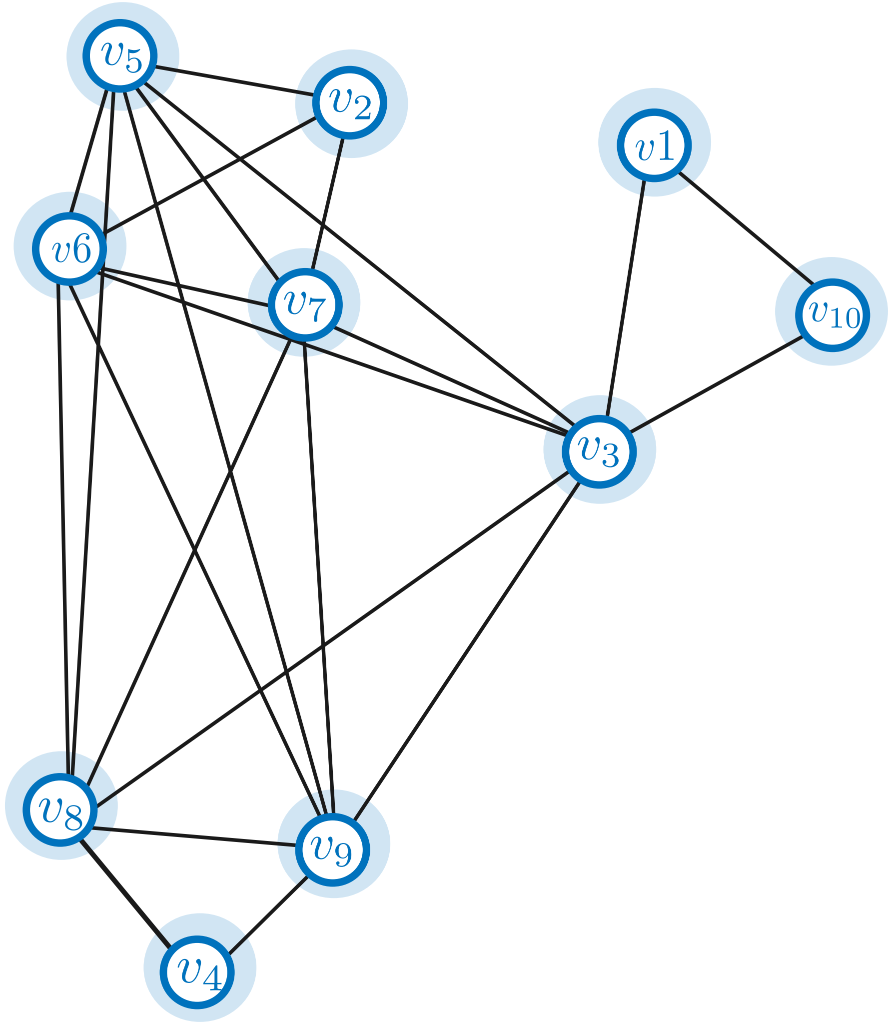
Evolution of the probability distribution from each node

Clustering based on similarity of the resulting **trajectories**

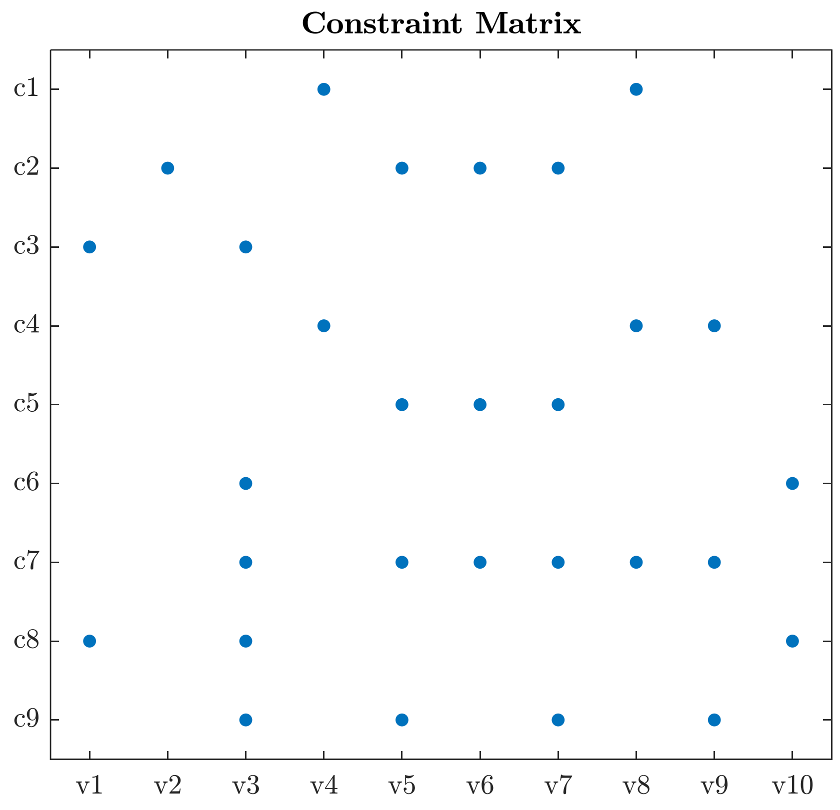
Retrieval of the associated **permuted matrix**

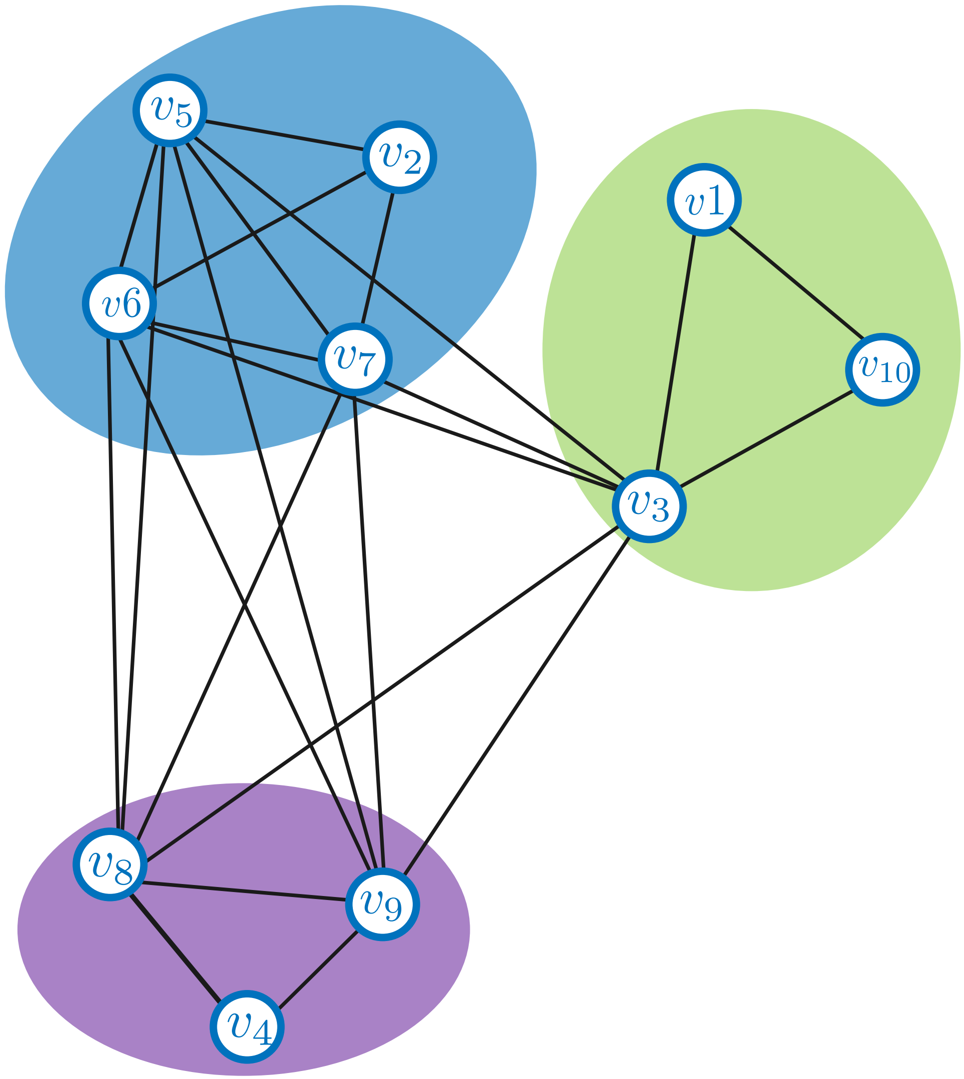
Decomposition strategies



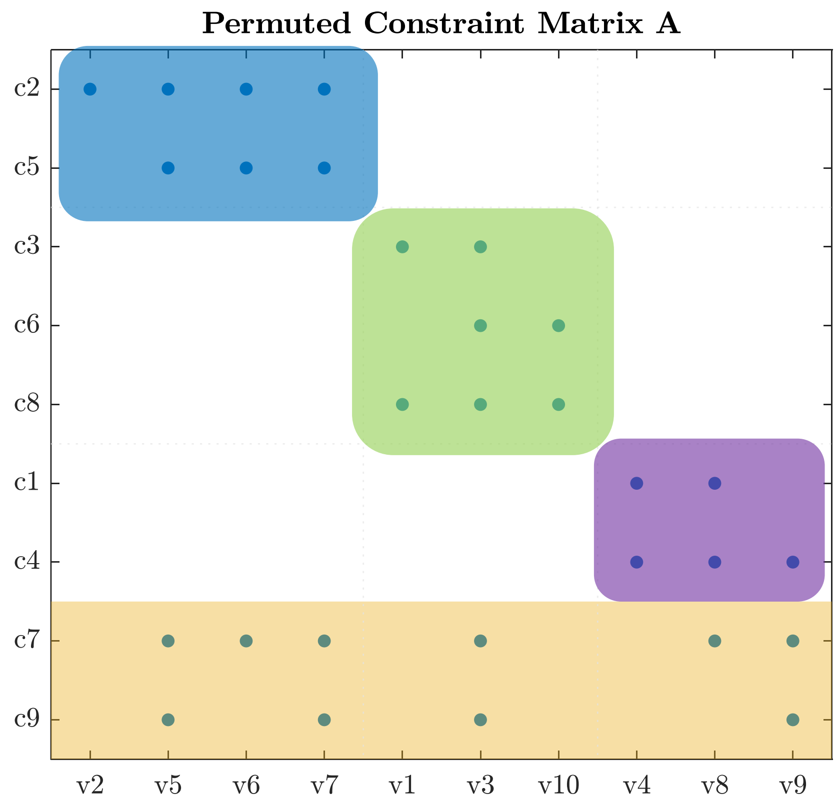


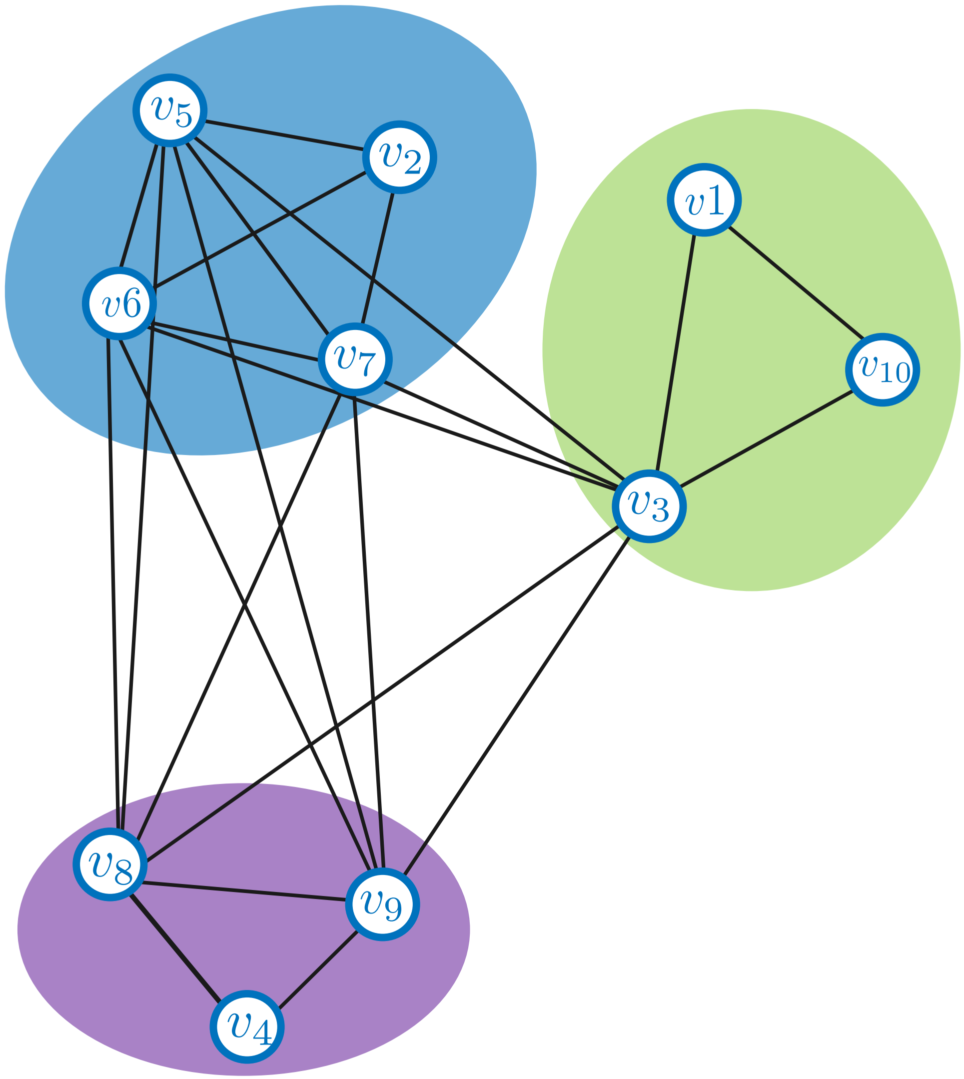
Decomposition strategies



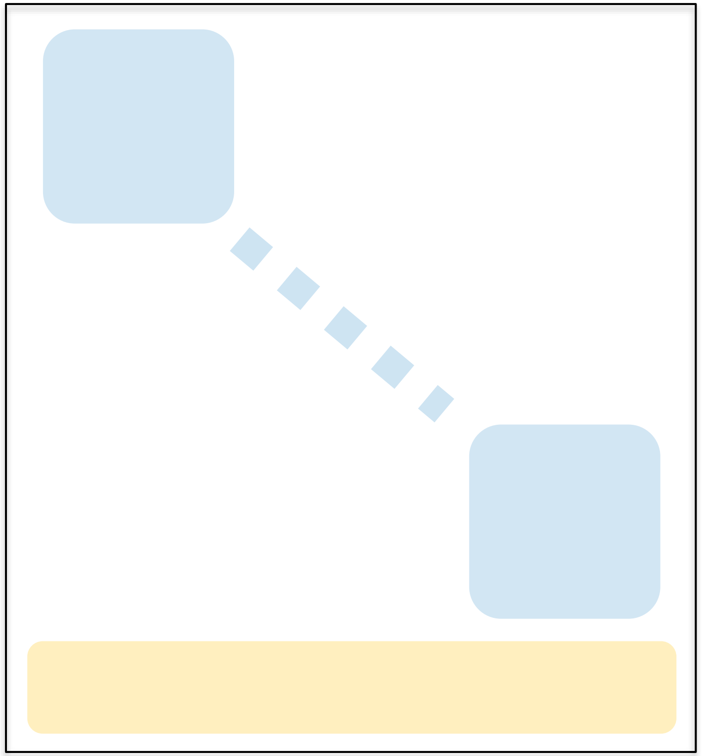


Decomposition strategies



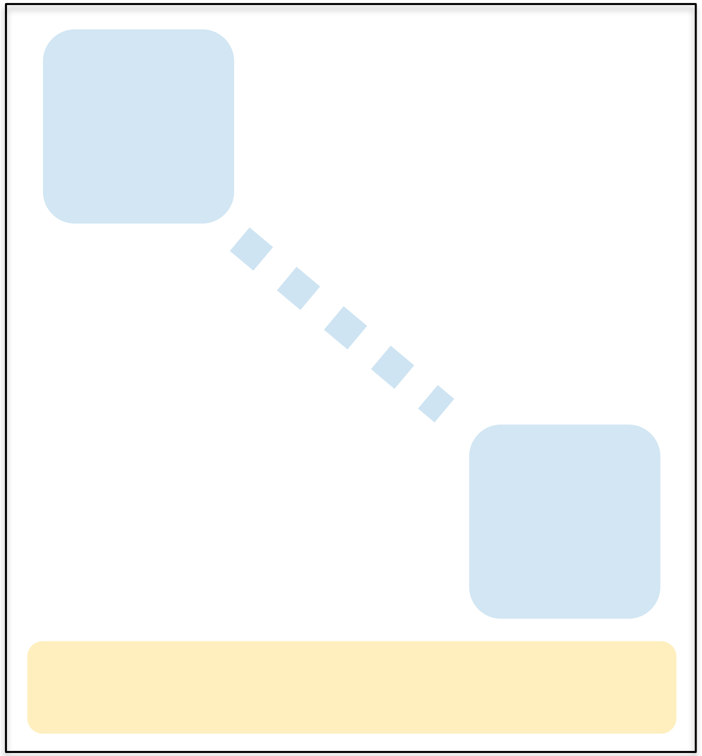


Singly-Bordered block-diagonal matrix

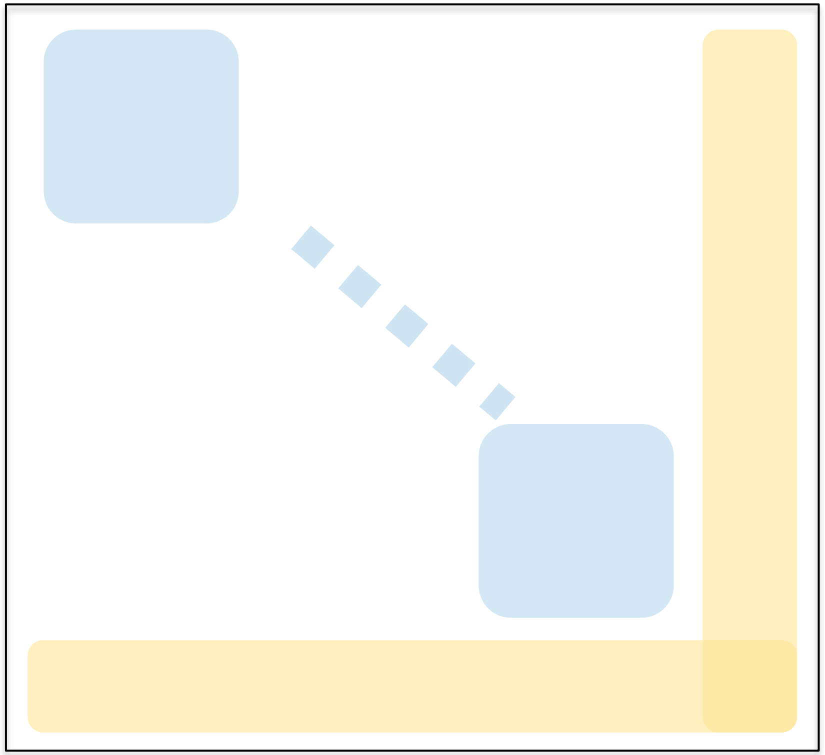


Decomposition strategies

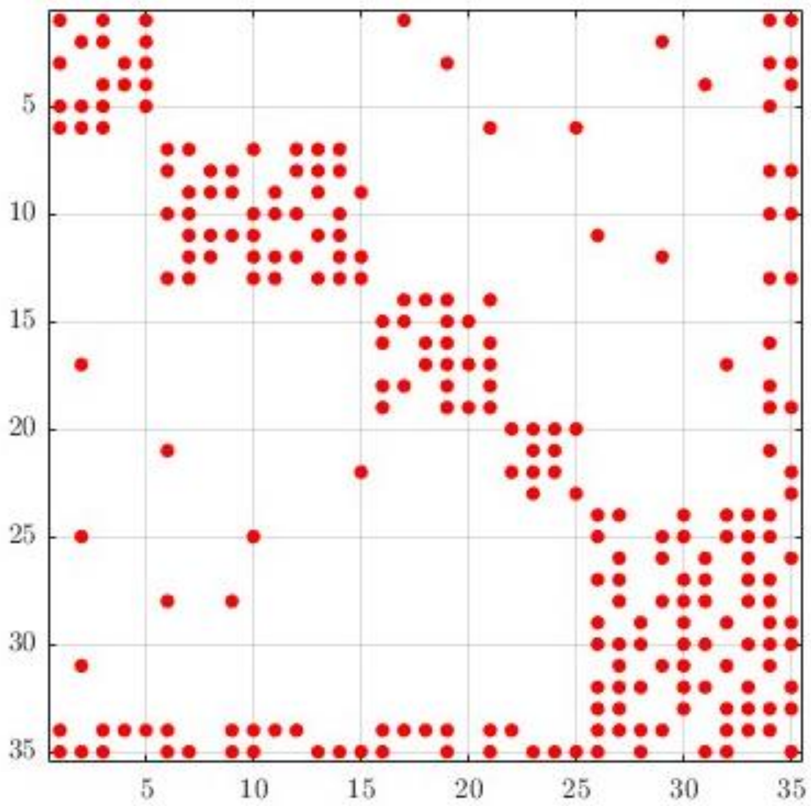
Singly-Bordered block-diagonal matrix



Doubly-Bordered block-diagonal matrix



Decomposition strategies



Modular architecture design in Systems Engineering

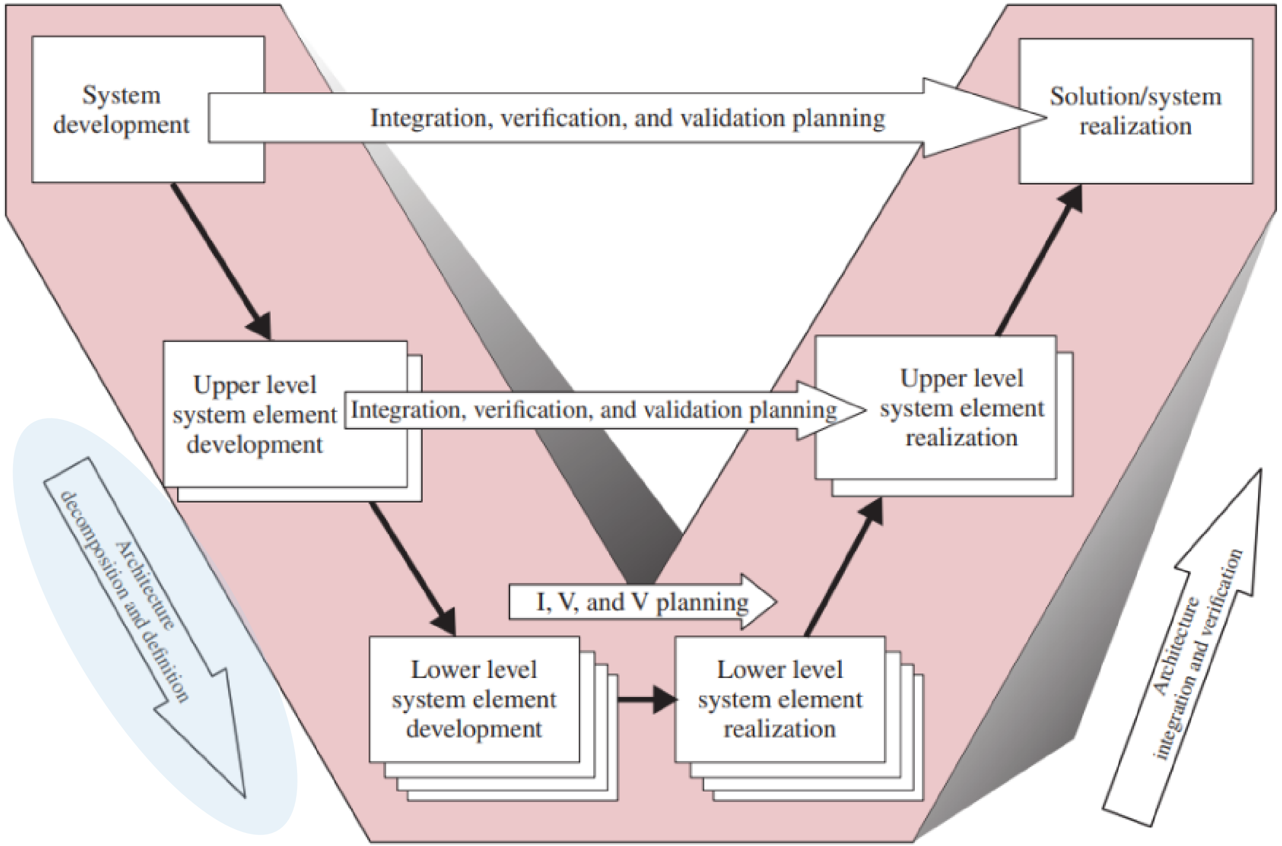


Image from INCOSE Systems Engineering Handbook, 2015

Beatrice Melani, et al. **"Logical Architecture Optimization via a Markov chain based Hierarchical Clustering Method."** 34th Annual INCOSE International Symposium, Dublin, Ireland, 2-6 July 2024.

Resolution schemes for multi-agent MILPs

Constraint-coupled multi-agent MILP

$$
\begin{aligned}
\min_{x_1,\dots,x_m} \quad & \sum_{i=1}^{m} c_i^\top x_i \\
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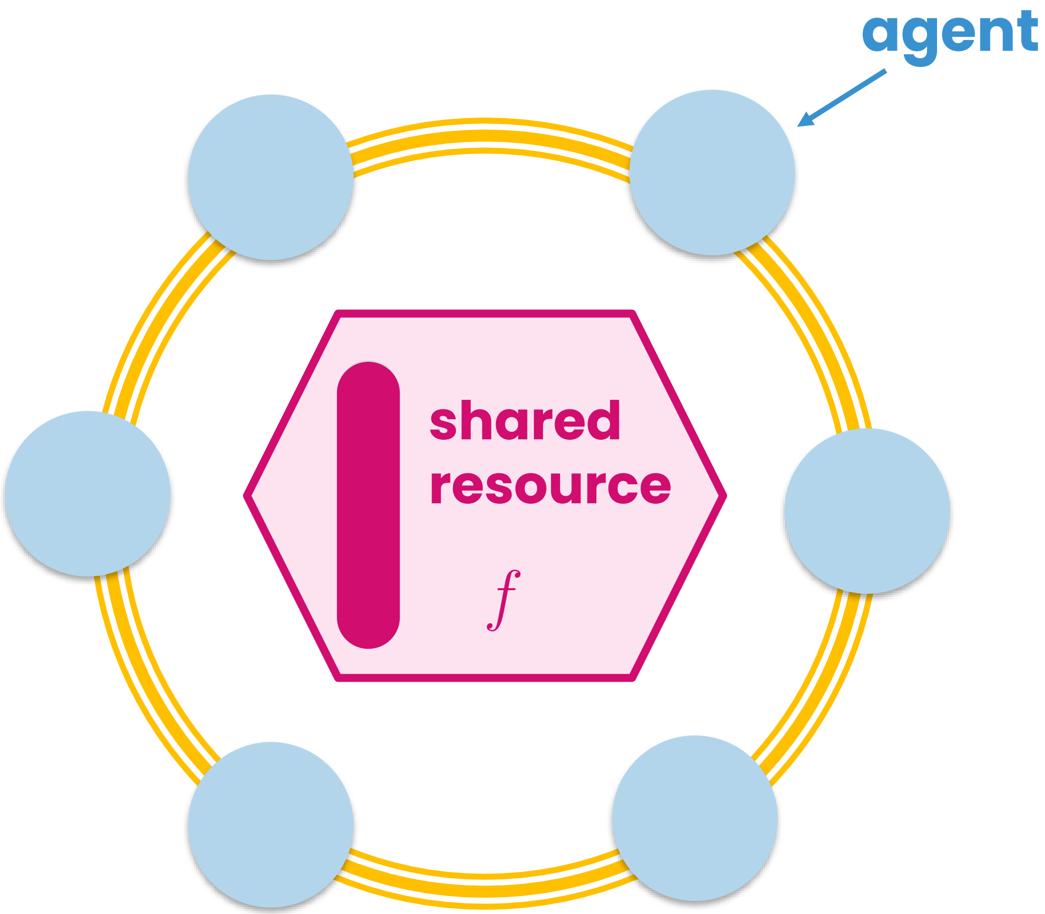
\end{aligned}
$$

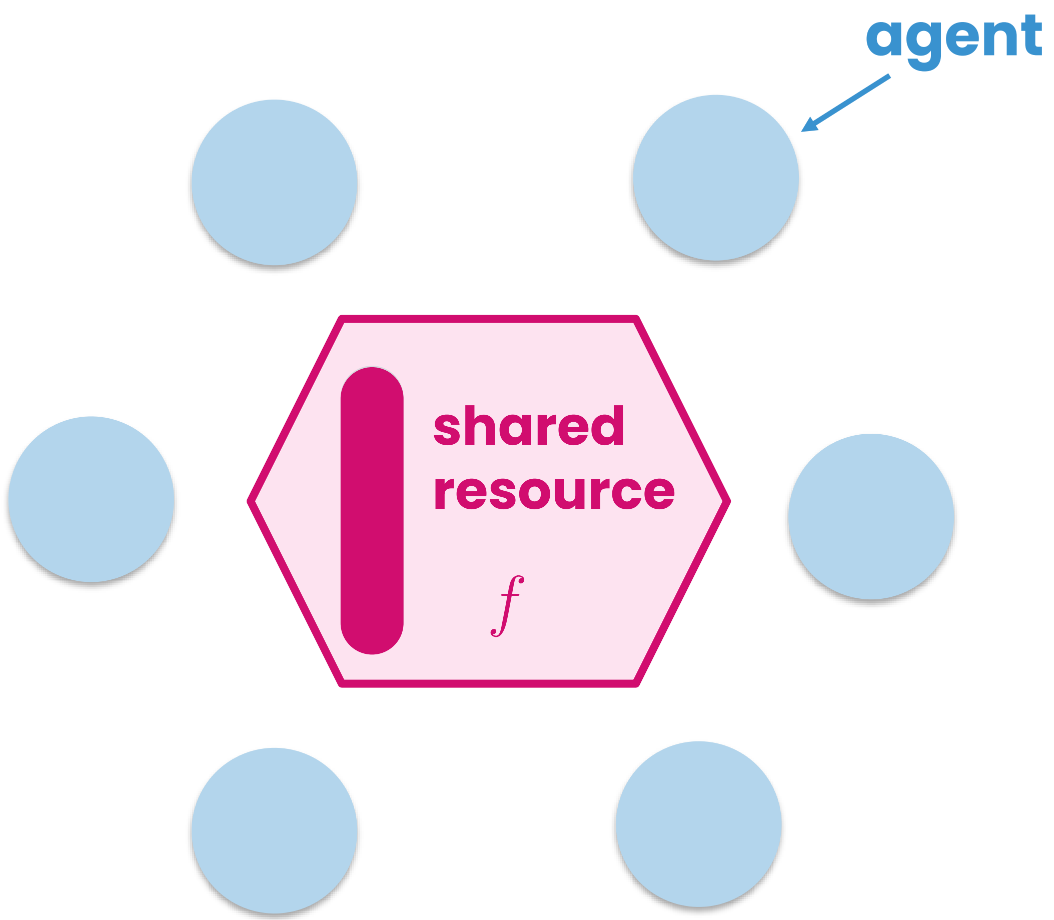
Decentralized iterative resolution schemes resorting to

Lagrangian relaxation



Decentralized iterative resolution schemes resorting to

Lagrangian relaxation



Primal problem

$$\mathcal{P}: \quad \min_{x_1,\dots,x_m} \quad \sum_{i=1}^{m} c_i^\top x_i + \lambda^\top \left(\sum_{i=1}^{m} E_i x_i - f \right)$$

subject to:

$$x_i \in X_i$$

$$\cancel{\sum_{i=1}^{m} E_i x_i \leq f}$$

Lagrange multipliers

Dual problem

$$\mathcal{D}: \quad \max_{\lambda \geq 0} -\lambda^\top f + \sum_{i=1}^{m} \min_{x_i \in X_i} (c_i^\top + \lambda^\top E_i) x_i \quad \Rightarrow \quad \lambda^\star$$

Convex problem providing a lower bound on the cost

Can be solved via **decentralized sub-gradient algorithm**

Dual problem

$$\mathcal{D}: \quad \max_{\lambda \geq 0} -\lambda^\top f + \sum_{i=1}^{m} \min_{x_i \in X_i} (c_i^\top + \lambda^\top E_i) x_i$$

Decentralized sub-gradient algorithm

$$x_i(\lambda(k)) \in \arg \min_{x_i \in X_i} (c_i^\top + \lambda(k)^\top E_i) x_i \qquad i = 1, \dots, m$$

$$\lambda(k+1) = \left[\lambda(k) + \alpha(k) \left(\sum_{i=1}^{m} E_i x_i(\lambda(k)) - f \right) \right]_+$$

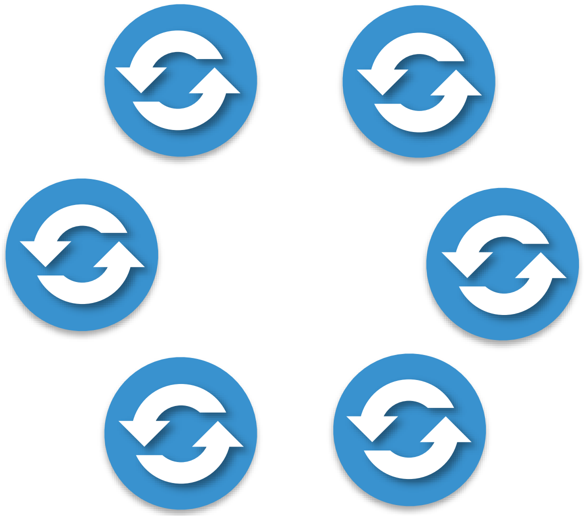
Decentralized resolution schemes

Dual problem

$$\mathcal{D}: \quad \max_{\lambda \geq 0} -\lambda^\top f + \sum_{i=1}^{m} \min_{x_i \in X_i} (c_i^\top + \lambda^\top E_i) x_i$$

$x_i(\lambda_k)$

Each **agent** solves a lower-dimensional problem and computes $x_i(\lambda_k)$



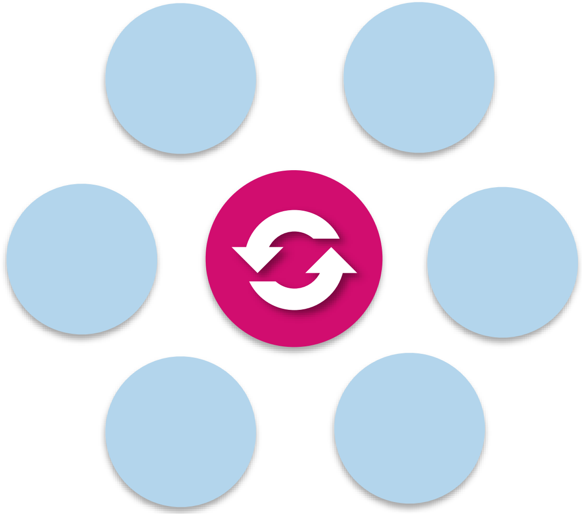
Decentralized resolution schemes

Dual problem

$$\mathcal{D}: \quad \max_{\lambda \geq 0} -\lambda^\top f + \sum_{i=1}^{m} \min_{x_i \in X_i} (c_i^\top + \lambda^\top E_i) x_i \quad \Rightarrow \quad \lambda^\star$$

Each **agent** solves a lower-dimensional problem and computes $\boldsymbol{x_i(\lambda_k)}$

A **central unit** updates $\boldsymbol{\lambda_k}$ based on $E_i x_i(\lambda_k)\ \forall i$



Dual problem

$$\mathcal{D}: \quad \max_{\lambda \geq 0} -\lambda^\top f + \sum_{i=1}^{m} \min_{x_i \in X_i} (c_i^\top + \lambda^\top E_i) x_i \Rightarrow \lambda^\star \Rightarrow x(\lambda^\star)$$

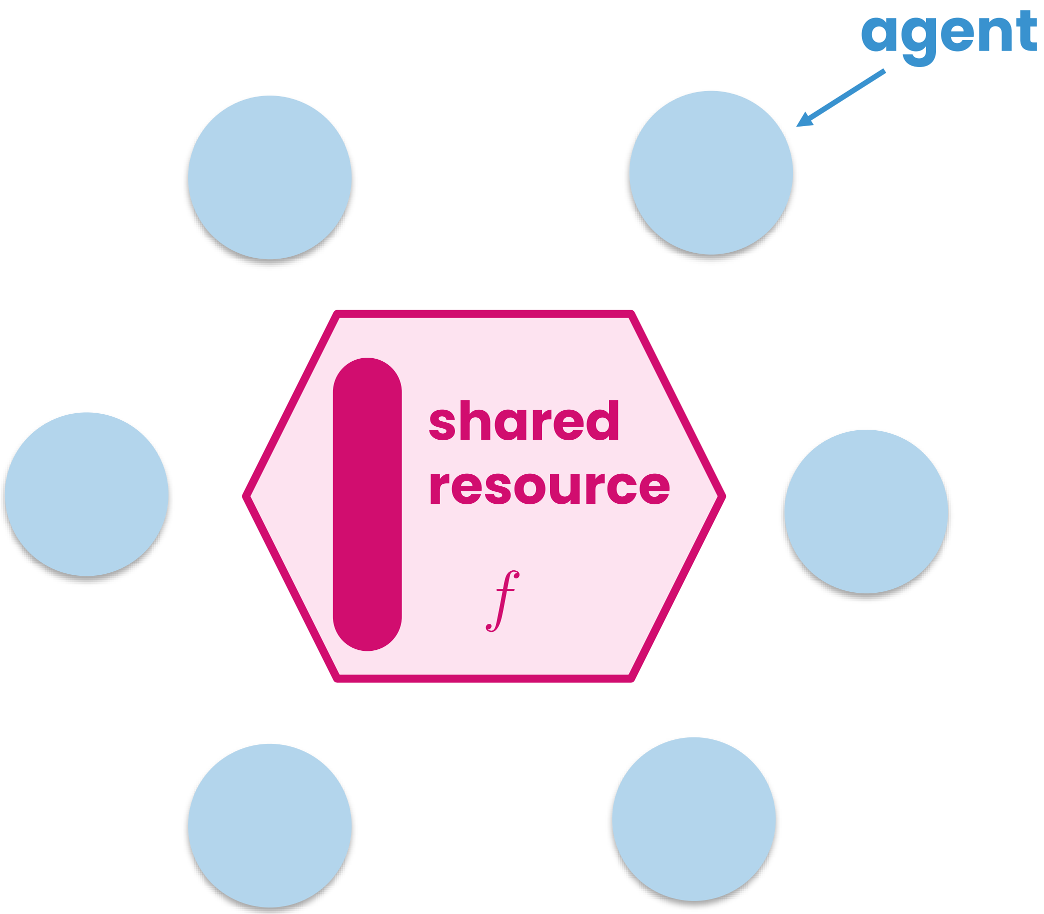
Optimal $\lambda^\star$ via **distributed lower dimensional computations**

Recovered $x(\lambda^\star)$ may **not satisfy** the **coupling constraints**

Decentralized resolution schemes

Decentralized iterative resolution schemes resorting to

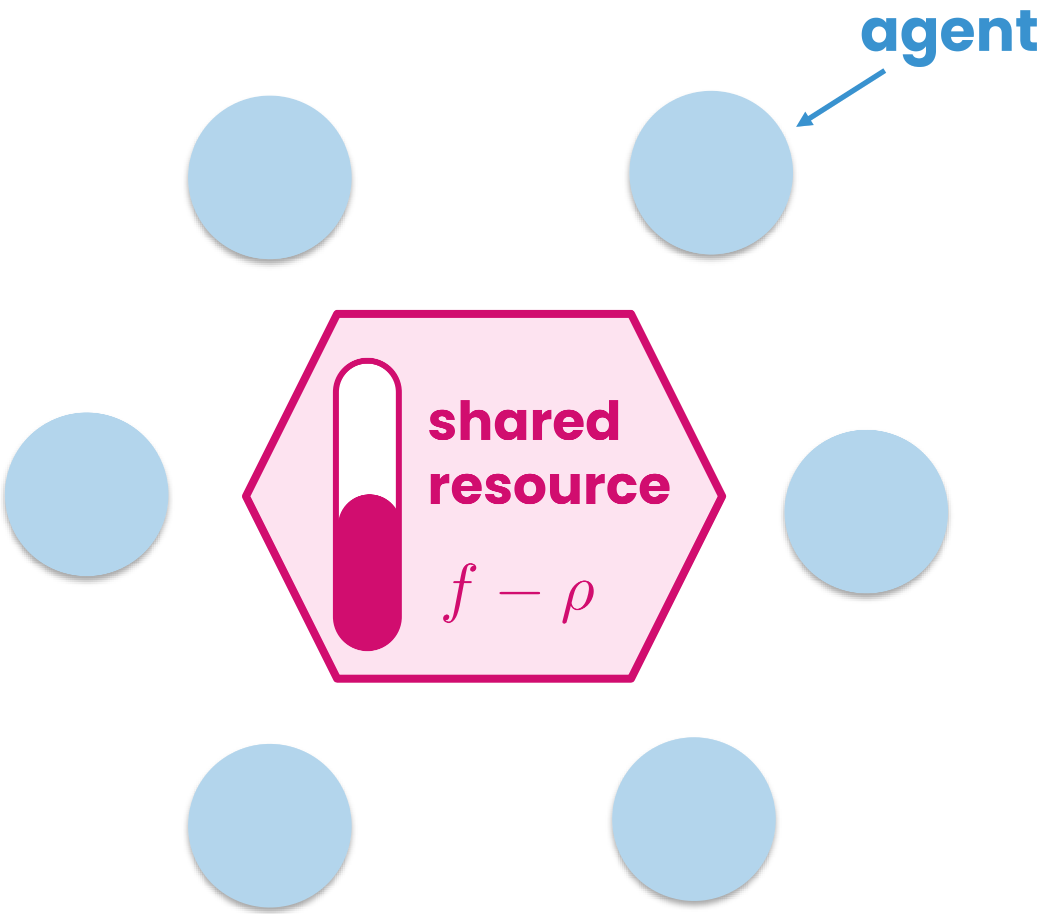
Lagrangian relaxation



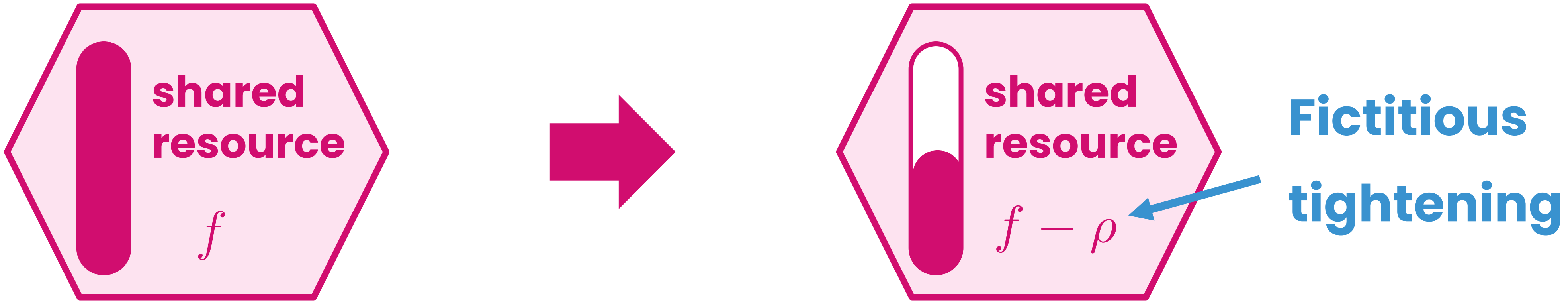
Decentralized resolution schemes

Decentralized iterative resolution schemes resorting to

- **Lagrangian relaxation**
- **Resource tightening**



Decentralized resolution schemes



How to compute the **tightening** to enforce **satisfaction** of the **coupling constraints?**

Sufficient condition

$$\rho \geq v(x(\lambda_\rho^\star)) \;\Rightarrow\; x(\lambda_\rho^\star) \quad \text{satisfies the coupling constraints}$$

Sufficient implicit condition

$$\rho \geq v(x(\lambda_\rho^\star)) \quad \Rightarrow \quad x(\lambda_\rho^\star)$$ satisfies the coupling constraints

function **depending** on the **solution** of an optimization **problem**

Decentralized resolution schemes

Vujanic et Al.

A-priori **worst-case** upper-bound $\tilde{\boldsymbol{\rho}}$ based on **all the admissible solutions** of each agent.

$$\rho = \tilde{\rho} \geq v(x(\lambda_\rho^\star)) \quad \forall \rho$$

Vujanic Robin, et al. "**A decomposition method for large scale MILPs, with performance guarantees and a power system application.**" *Automatica 67 (2016): 144-156.*

Vujanic et Al.

A-priori **worst-case** upper-bound $\tilde{\rho}$ based on **all the admissible solutions** of each agent.

$$\tilde{\rho} = p \max_{i=1,\dots,m} \left\{ \max_{x_i \in X_i} E_i x_i - \min_{x_i \in X_i} E_i x_i \right\}$$

depending on **all** admissible **solutions**

Vujanic Robin, et al. **"A decomposition method for large scale MILPs, with performance guarantees and a power system application."** *Automatica 67 (2016): 144-156.*

Decentralized resolution schemes

Falsone et Al.

Non-decreasing **adaptive** tightening $\boldsymbol{\rho}_k$ based only on the tentative **solutions explored** by the resolution scheme.

$$\rho_{k+1} = \max_{t=0,\ldots,k} v(x(\lambda^\star_{\rho_t}))$$

Falsone Alessandro et Al., **"A decentralized approach to multi-agent MILPs: finite-time feasibility and performance guarantees."** *Automatica 103 (2019): 141-150.*

Decentralized resolution schemes

Falsone et Al.

Non-decreasing **adaptive** tightening $\boldsymbol{\rho}_k$ based only on the tentative **solutions explored** by the resolution scheme.

$$\rho_{k+1} = p \max_{i=1,\dots,m} \left\{ \max_{t=0,\dots,k} E_i x_{i,t} - \min_{t=0,\dots,k} E_i x_{i,t} \right\}$$

depending on **explored solutions**

Falsone Alessandro et al., **"A decentralized approach to multi-agent MILPs: finite-time feasibility and performance guarantees."** *Automatica 103 (2019): 141-150.*

Decentralized resolution schemes – Memoryless update

Vujanic et Al.

$$\rho = \tilde{\rho}$$

A priori bound based on **all** feasible **solutions**

Falsone et Al.

$$\rho_{k+1} = \max_{t=0,\ldots,k} v(x(\lambda^\star_{\rho_t}))$$

Adaptive bound based on all **explored solutions**

Both **conservative**, may be **inapplicable** or lead to **poor performance** (worsens as $\|\rho\|_\infty$ increases)

Memory-less update

Adaptive tightening $\boldsymbol{\rho}_k$ derived based on the **current** tentative **solution** obtained by the resolution scheme.

$$\rho_{k+1} = v(x(\lambda^{\star}_{\rho_k}))$$

Manieri Lucrezia et Al., **"A novel decentralized approach to large-scale multi-agent MILPs."** *IFAC-PapersOnLine 56 (2023)*: 5919-5924.

Decentralized resolution schemes – Memoryless update

Conservativity

Vujanic et Al.

$$\rho = \tilde{\rho}$$

A priori bound based on **all** feasible **solutions**

Falsone et Al.

$$\rho_{k+1} = \max_{t=0,\dots,k} v(x(\lambda^\star_{\rho_t}))$$

Adaptive bound based on all **explored solutions**

Memory-less update

$$\rho_{k+1} = v(x(\lambda_{\rho_k}))$$

Adaptive bound based on **current solution** only

Decentralized resolution schemes – Memoryless update

Memory-less update

Less conservative

No convergence guarantees

Manieri Lucrezia et Al., **"A novel decentralized approach to large-scale multi-agent MILPs."** *IFAC-PapersOnLine 56 (2023)*: 5919-5924.

Decentralized resolution schemes – Memoryless update

Memory-less update

Less conservative

No convergence guarantees

Integral update

Manieri Lucrezia et Al., **"Handling complexity in large-scale cyber-physical systems through distributed computation"** chapter in *Computation-Aware Algorithmic Design for Cyber-Physical Systems, Springer (2022)*

Decentralized resolution schemes – Integral update

ρ

$$\mathcal{D}_\rho \to \lambda^\star_\rho \to x(\lambda^\star_\rho)$$

Decentralized resolution schemes – Integral update

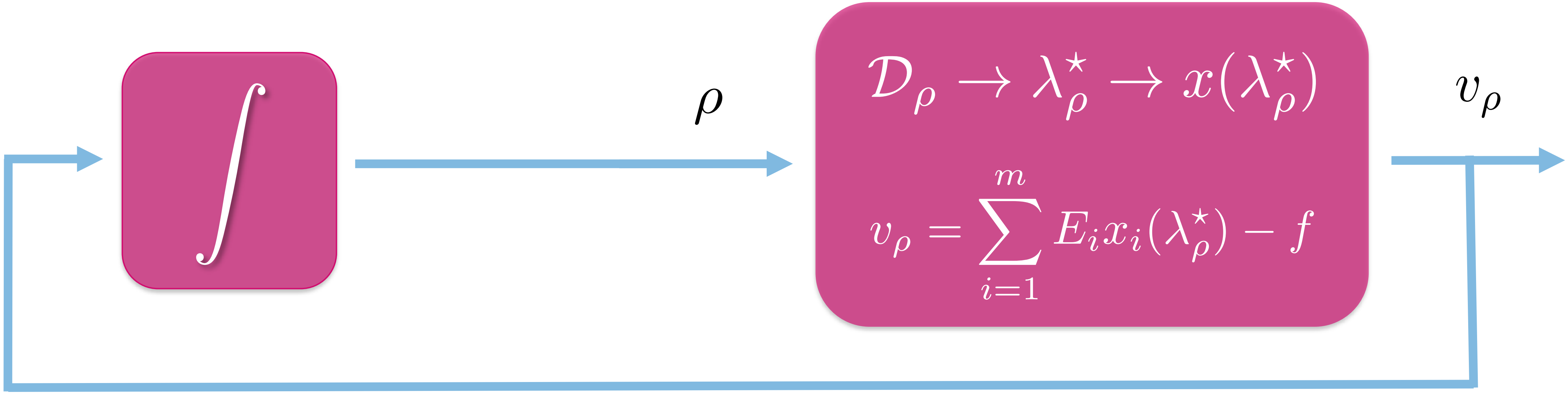
$$\rho$$

$$\mathcal{D}_\rho \to \lambda_\rho^\star \to x(\lambda_\rho^\star)$$

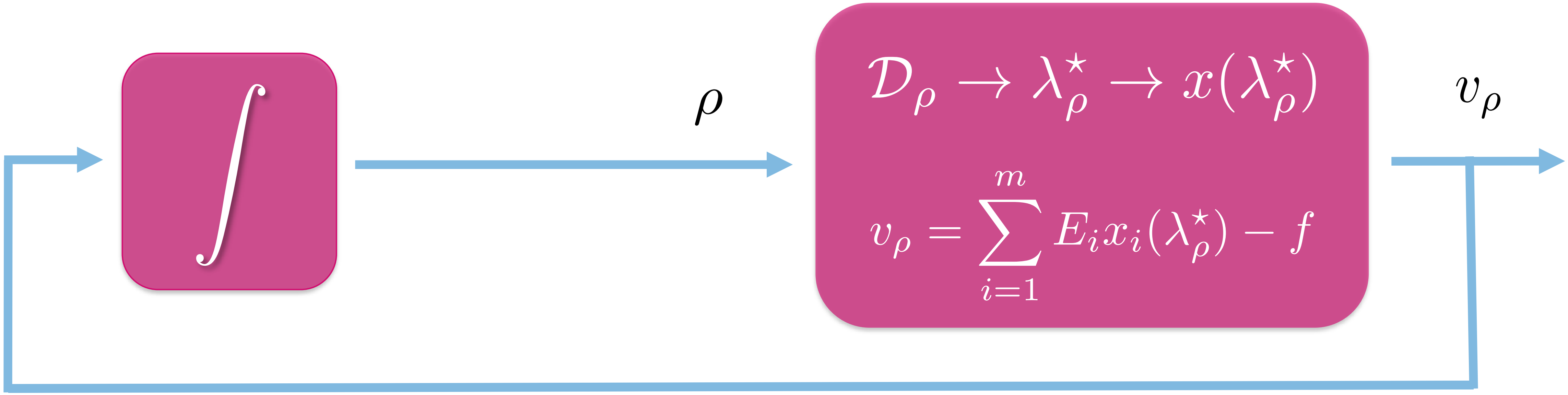
$$v_\rho = \sum_{i=1}^{m} E_i x_i(\lambda_\rho^\star) - f$$

$$v_\rho$$

Decentralized resolution schemes – Integral update

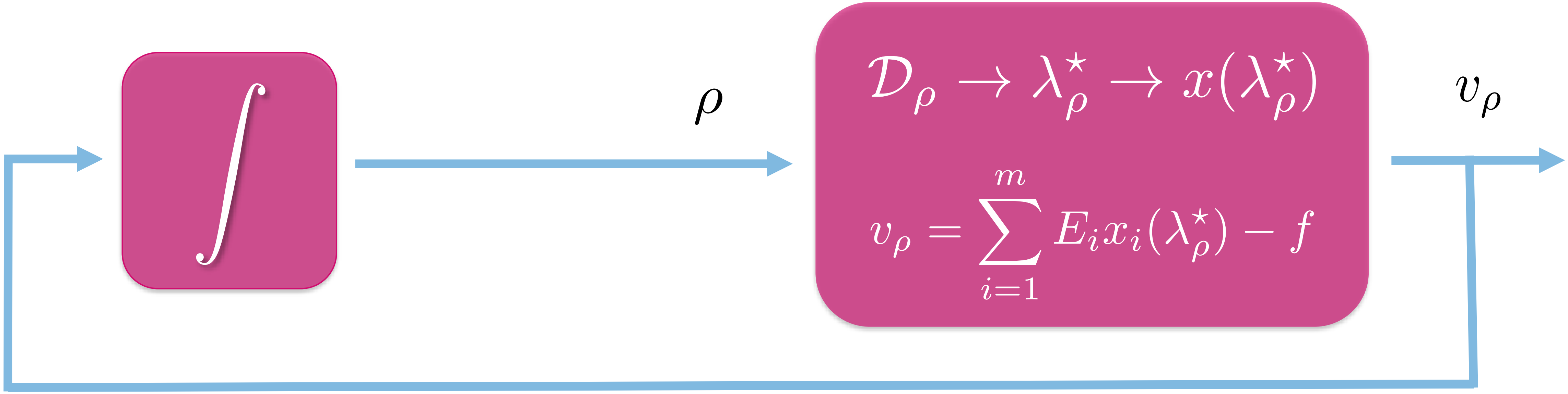


Decentralized resolution schemes – Integral update



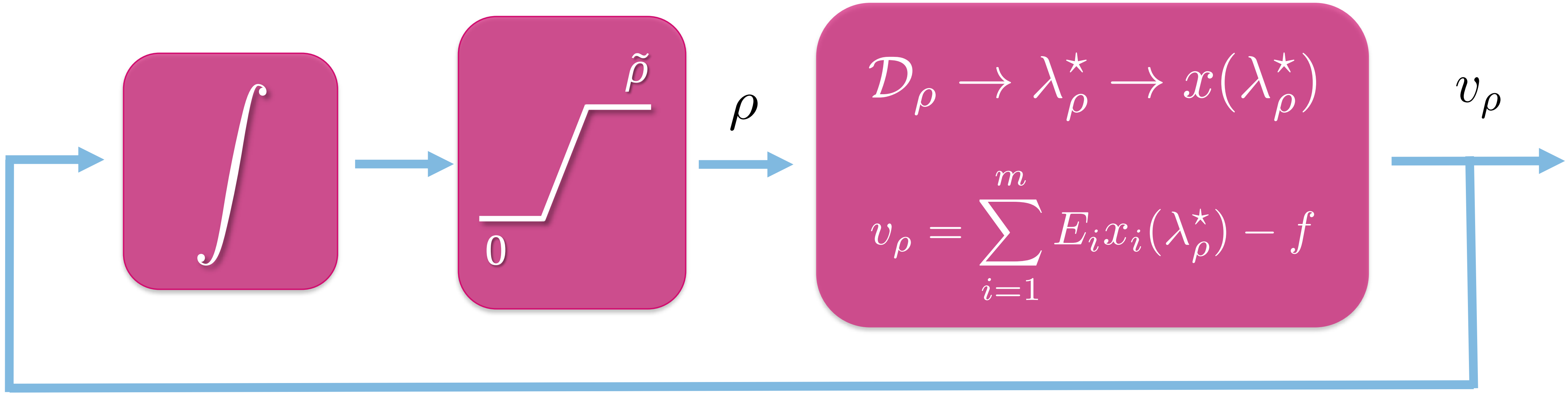
- As long as $\boldsymbol{x}(\boldsymbol{\lambda}_\rho^\star)$ is **unfeasible**, $v_\rho \geq 0$ and ρ **increases**

Decentralized resolution schemes – Integral update



- As long as $\boldsymbol{x}(\boldsymbol{\lambda}_\rho^\star)$ is **unfeasible**, $v_\rho \geq 0$ and ρ **increases**
- $\mathcal{D}_\rho$ enforces coupling constraints→ v_ρ is **small**

Decentralized resolution schemes – Integral update



- As long as $\boldsymbol{x}(\boldsymbol{\lambda}_{\boldsymbol{\rho}}^\star)$ is **unfeasible**, $v_\rho \geq 0$ and ρ **increases**
- $\mathcal{D}_\rho$ enforces coupling constraints→ v_ρ is **small**
- **Saturation** of ρ to $\tilde{\boldsymbol{\rho}}$

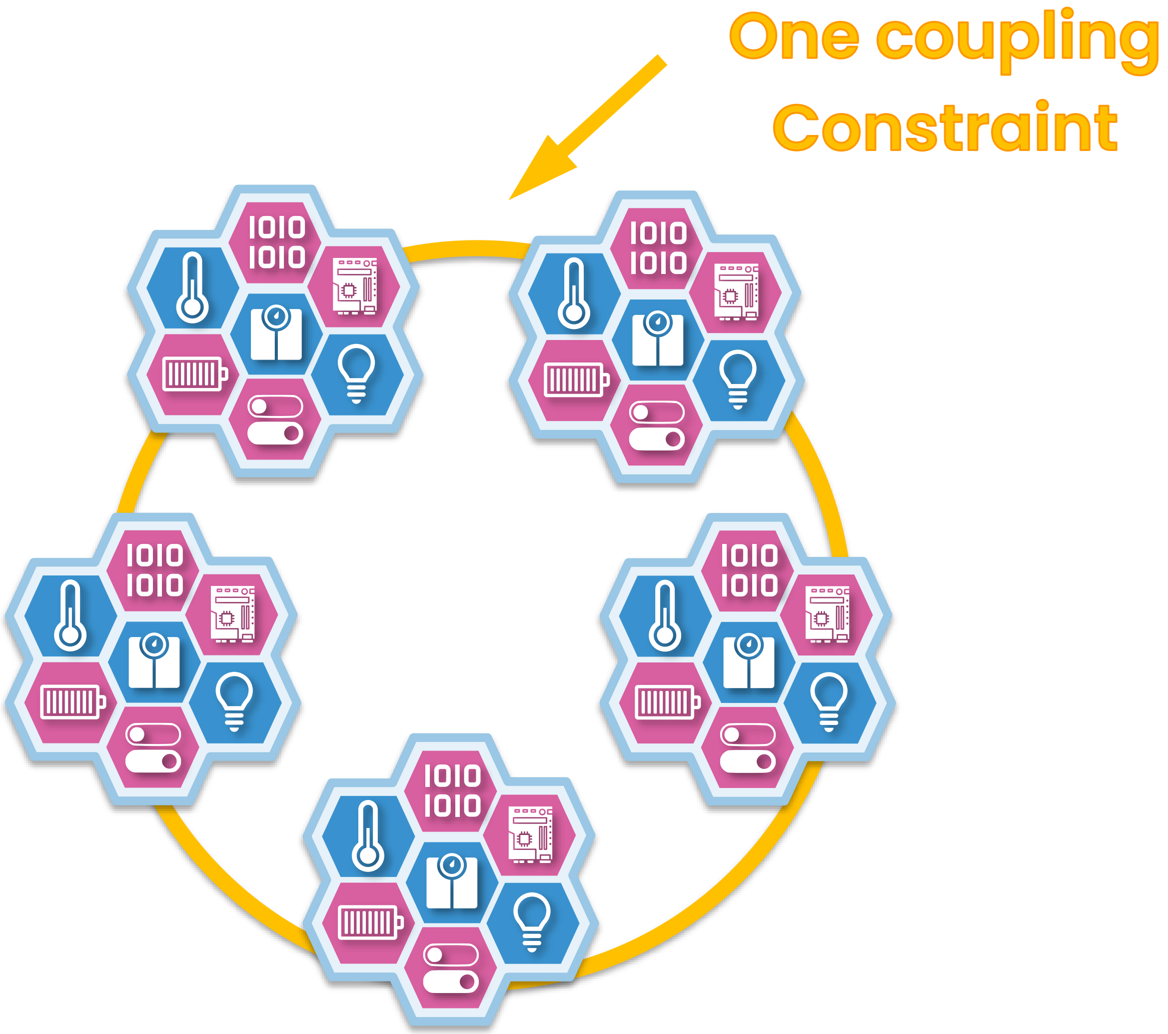
Decentralized resolution schemes – Memoryless update

Memory-less update	*Integral update*
No convergence guarantees	Guaranteed **convergence** to a **feasible** solution
Less conservative	**Less conservative**

Manieri Lucrezia et Al., **"A novel decentralized approach to large-scale multi-agent MILPs."** *IFAC-PapersOnLine 56 (2023)*: 5919-5924.

Manieri Lucrezia et Al., **"Handling complexity in large-scale cyber-physical systems through distributed computation"** chapter in *Computation-Aware Algorithmic Design for Cyber-Physical Systems, Springer (2022)*

Decentralized resolution schemes – Scalar coupling



Dual Bisection Algorithm

- **No tightening** needed
- **Bisection-based**
- **Finite-time convergence to a feasible solution**

Manieri Lucrezia et Al., " **Dualbi: A dual bisection algorithm for non-convex problems with a scalar complicating constraint** "
Submitted to Automatica for possible publication and available on arXiv

Decentralized resolution schemes – Scalar coupling

Non-convex cost function

$$\min_{x} \; f(x) \quad \text{s.t.} \quad x \in X, \quad v(x) \leq 0$$

"Simple" constraints

Single complicating constraint

Dual Bisection Algorithm

- **No tightening** needed
- **Bisection-based**
- **Finite-time convergence to a feasible solution**

Manieri Lucrezia et Al., " **Dualbi: A dual bisection algorithm for non-convex problems with a scalar complicating constraint** "
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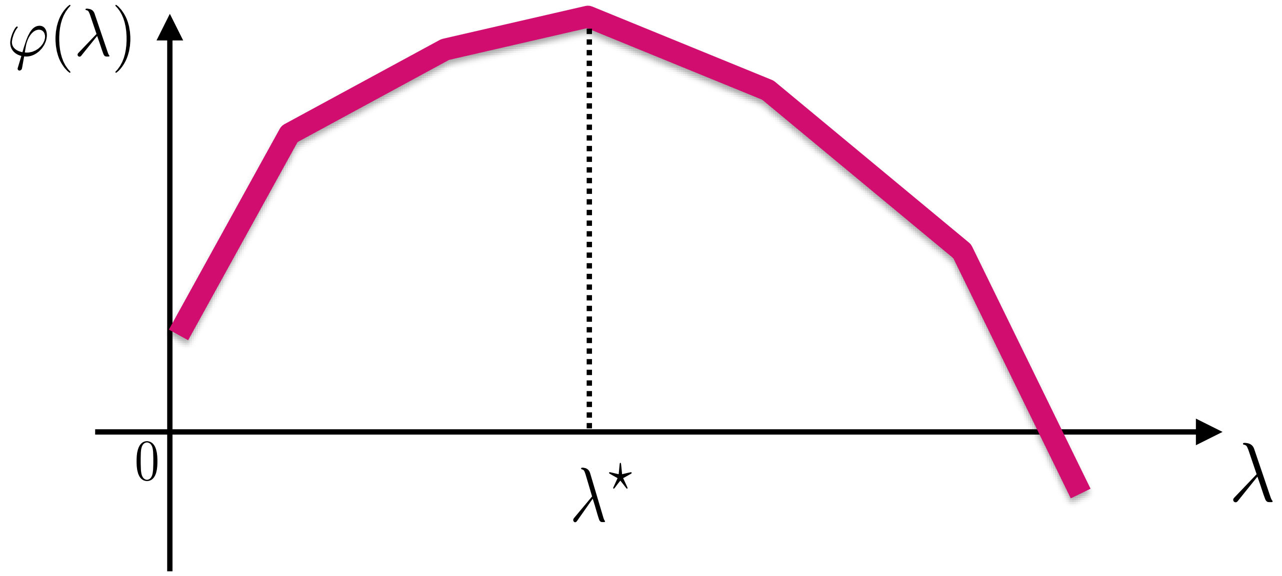
Solving non-convex problems with a complicating constraint

$$\mathcal{P}: \min_{x} \quad f(x)$$
$$\text{s.t.} \quad x \in X$$
$$v(x) \leq 0$$

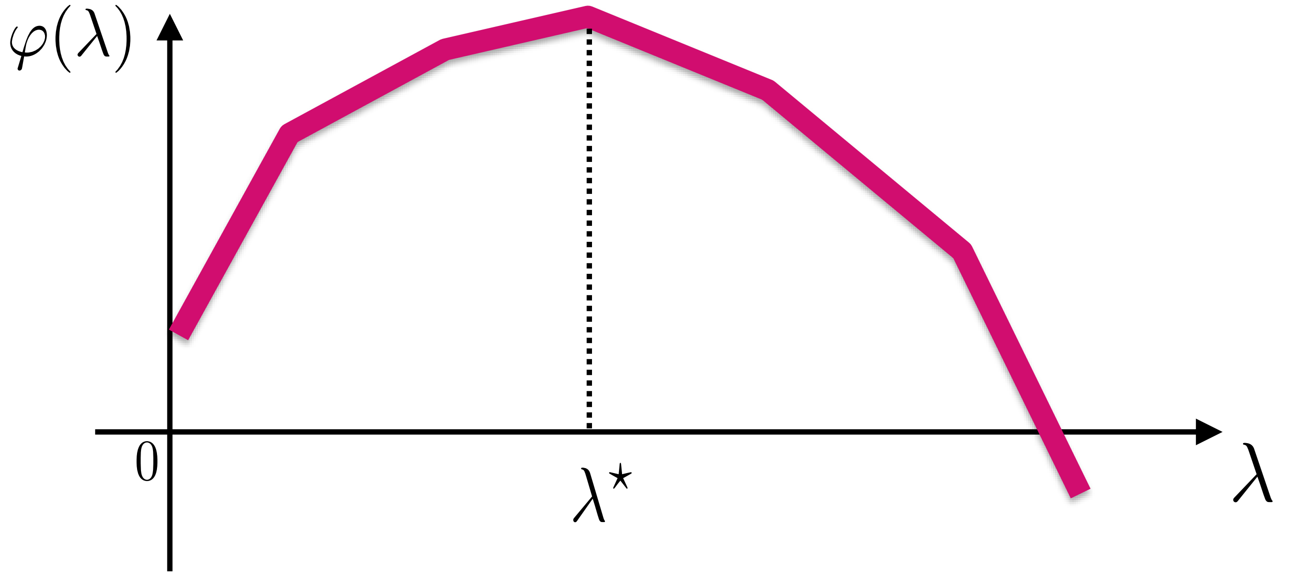


$$\mathcal{D}: \max_{\lambda \geq 0} \underbrace{\min_{x \in X} f(x) + \lambda v(x)}_{\varphi(\lambda)}$$

The **dual function** is **scalar and concave**

Solving non-convex problems with a complicating constraint

$$\mathcal{P}: \min_{x} \quad f(x)$$
$$\text{s.t.} \quad x \in X$$
$$v(x) \leq 0$$

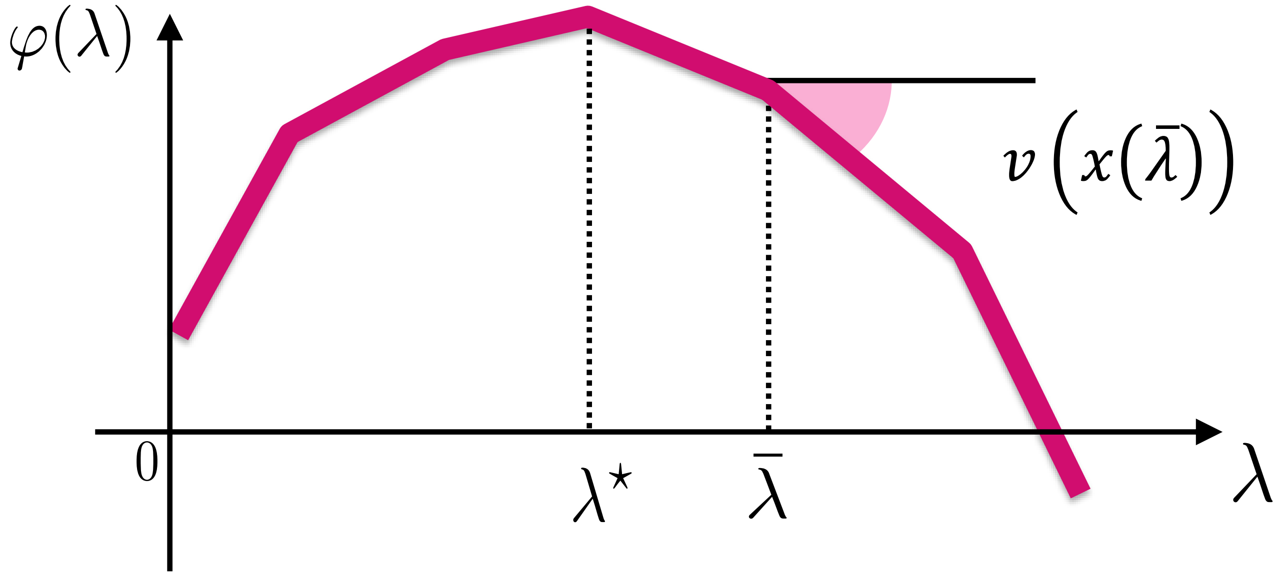


$$\mathcal{D}: \max_{\lambda \geq 0} \underbrace{\min_{x \in X} f(x) + \lambda v(x)}_{\varphi(\lambda)}$$

Dual problem solved via a bisection method to find a zero of the sub-differential

Solving non-convex problems with a complicating constraint

$$\mathcal{P}: \min_{x} \quad f(x)$$
$$\text{s.t.} \quad x \in X$$
$$v(x) \leq 0$$



$$\mathcal{D}: \max_{\lambda \geq 0} \underbrace{\min_{x \in X} f(x) + \lambda v(x)}_{\varphi(\lambda)}$$

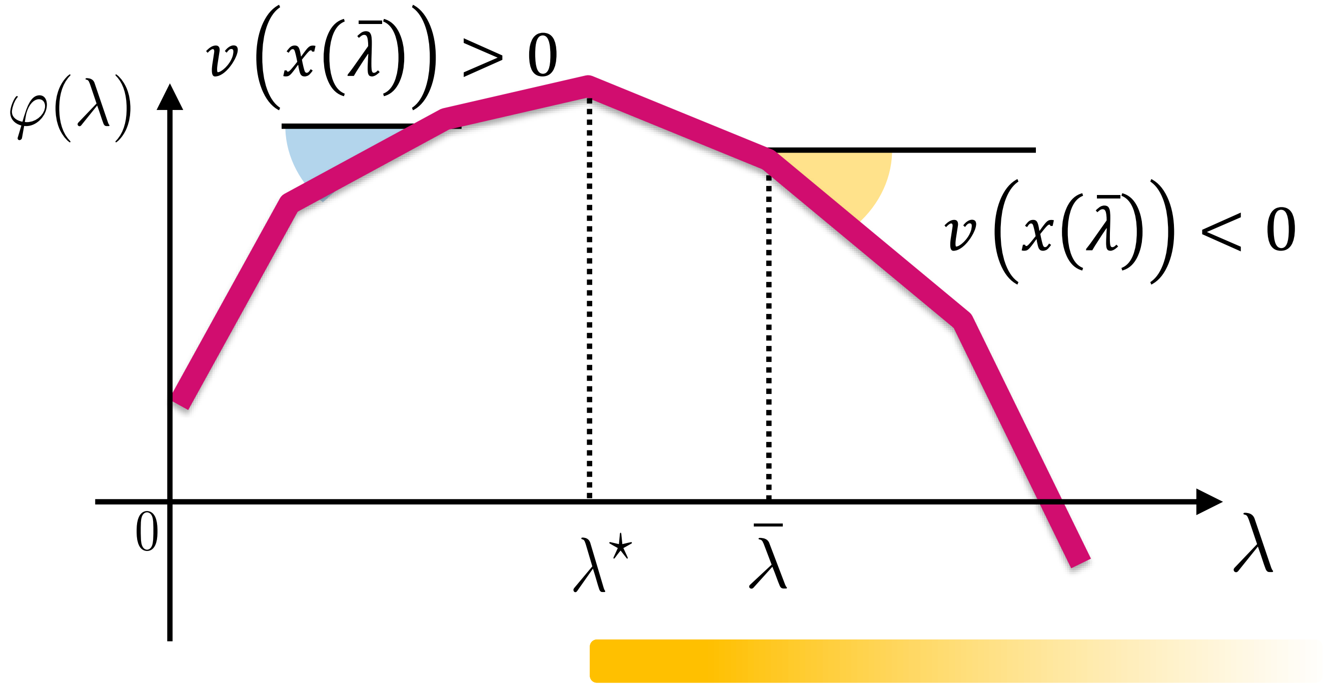
$$x(\lambda) = \arg\min_{x \in X} f(x) + \lambda v(x)$$

Solving non-convex problems with a complicating constraint

$$\mathcal{P}: \min_{x} \quad f(x)$$
$$\text{s.t.} \quad x \in X$$
$$v(x) \leq 0$$

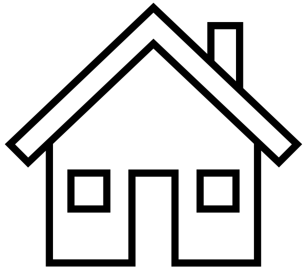
$$\mathcal{D}: \max_{\lambda \geq 0} \underbrace{\min_{x \in X} f(x) + \lambda v(x)}_{\varphi(\lambda)}$$

$$x(\lambda) = \arg\min_{x \in X} f(x) + \lambda v(x)$$



$x(\bar{\lambda})$ **feasible**

Application to the provision of balancing services to the power grid



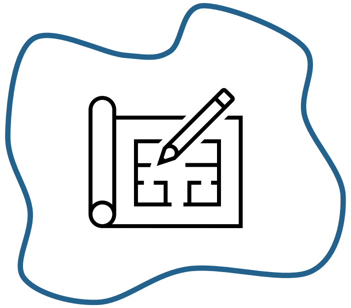
Prosumer

- Controllable generator G : $P_i^G > 0$
- Programmable load L : $P_i^L < 0$

  Assumed to work on n_i^L levels
- Battery Storage Device B : $P_i^B \lessgtr 0$
- Reference daily profile $\tilde{P}_i$

Pool

- m prosumers
- Power exchanged with the grid $P = \sum_{i=1}^{m} \left(P_i^G + P_i^B + P_i^L \right)$



BSP

One-day time horizon → M slots of duration τ_s

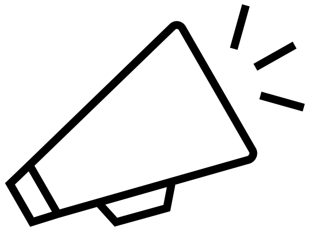
- $(t\tau_s, (t+1)\tau_s)$, $t \in \{0, \dots, M-1\}$

Balancing services provision via prosumers aggregation

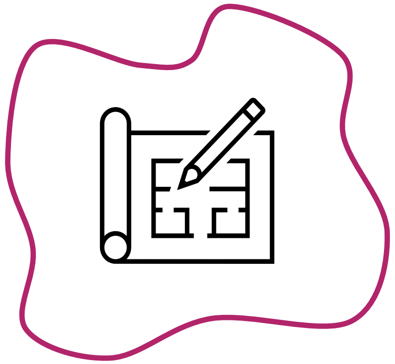
TSO

Variation of the power profile: $\Gamma(t)(1 \pm \varepsilon)$ $\varepsilon \in (0,1)$

In the time-interval $t_0, \dots, t_f$



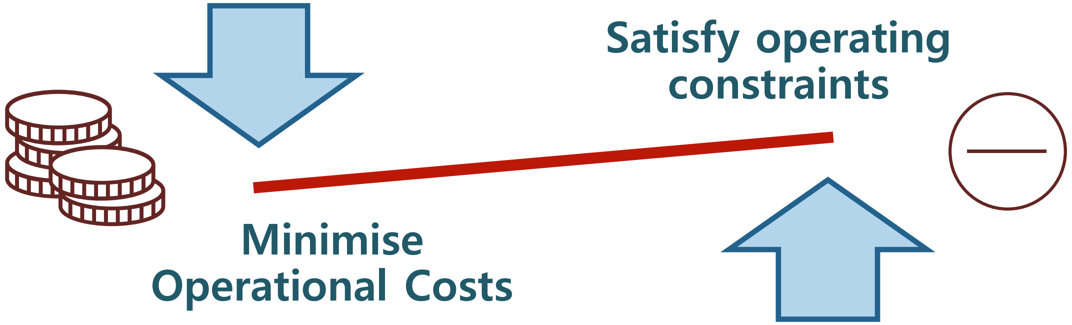
BSP



Re-distributes the request among all prosumers

$$\Gamma(t)(1-\varepsilon) \leq P(t) - \tilde{P}(t) \leq \Gamma(t)(1+\varepsilon) \qquad \forall t \in [t_0, t_f]$$

$$P_i(t) = \widetilde{P_i} \qquad \forall t \in [t_f + 1, M - 1]$$



MLD model

Variables (of the i^{th} prosumer)

$P_i^G > 0$

$\delta_i^G \in \{0,1\}$

$P_i^B \lessgtr 0$

$S_i(t)$

$P_i^L \in \left\{ 0, \ \frac{\bar{P}_i^L}{n_i^L}, \ 2\frac{\bar{P}_i^L}{n_i^L} \ \dots \ \bar{P}_i^L \right\}$

Assumption:

$n_i^L = 2^{J_i^L} - 1 \,, \quad J_i^L \in \mathbb{N}$

$\delta_{i,j}^L(t) \in \{0,1\} \qquad j = 1, \dots, J_i^L$

$$P_i^L = \sum_{j=1}^{J_i^L} \left(2_i^{j-1} \bar{P}_i^L \cdot \delta_{i,j}^L(t) \right) = \sigma^\top \cdot \delta_i^L(t)$$

$$\delta_i^L(t) = \begin{bmatrix} \delta_{i,1}^L \\ \vdots \\ \delta_{i,J_i^L}^L \end{bmatrix}$$

MLD model

Variables (of the i^{th} prosumer)

$P_i^G > 0$

$\delta_i^G \in \{0,1\}$

$P_i^B \lessgtr 0$

$S_i(t)$

$\delta_i^L(t) \in \{0,1\}^{J_i^L}$

State Vector

$$s(t) = \begin{bmatrix} S_1(t) \\ \vdots \\ S_m(t) \end{bmatrix}$$

Input Vector

$$u(t) = \begin{bmatrix} u_1(t) \\ \vdots \\ u_m(t) \end{bmatrix}$$

$$u_i(t) = \begin{bmatrix} u_{i,c}(t) \\ u_{i,d}(t) \end{bmatrix} = \begin{bmatrix} P_i^G(t) \\ P_i^B(t) \\ \delta_i^G(t) \\ \delta_i^L(t) \end{bmatrix}$$

MLD model

Operating Constraints (of the i^{th} prosumer)

Battery Storage Dynamics

$$S_i(t+1) = S_i(0) - \tau_s \sum_{s=0}^{t} P_i^B(s)$$
$$= S_i(t) - \tau_s P_i^B(t)$$

Energy Consumed by L

$$\sum_{t=t_0}^{M-1} \tau_s P_i^L(t) = \sum_{t=t_0}^{M-1} \tau_s \sigma^\top \delta_i^L(t) = E_i^L$$

Min/Max Capacity Level

$$\underline{S}_i \leq S_i(t) \leq \overline{S}_i$$

Min/Max Power Produced by G

$$\delta_i^G(t)\underline{P}_i^G \leq P_i^G(t) \leq \delta_i^G(t)\overline{P}_i^G$$

Charging/Discharging rates

$$P_i^{B,c} \leq P_i^B(t) \leq P_i^{B,d}$$

MLD model

Rescheduling Problem Constraints

Flexibility Limitation of L

$$P_i^L(t) = \sigma^\top \delta_i^L(t) = \widetilde{P}_i^L(t)$$

$$t < t_i^{L,0} \vee t > t_i^{L,f}$$

Rebound Effect Avoidance

$$P_i(t) = P_i^G(t) + \sigma^\top \delta_i^L(t) + P_i^B(t) = \widetilde{P}_i(t),$$

$$t = t_f + 1, \dots, M - 1$$

Power Variation (TSO request)

$$(1-\varepsilon)\Gamma(t) \leq \sum_{i=1}^{m} \left(P_i^G(t) + \sigma^\top \delta_i^L(t) + P_i^B(t)\right) - \widetilde{P}(t) \leq (1+\varepsilon)\Gamma(t),$$

$$t = t_0, \dots, t_f$$

MILP formulation

Operational Costs

$C_i^G > 0$: Unitary cost of the energy produced by G

$C_i^B > 0$: Unitary cost of the aging of battery B

$C_i^L > 0$: Unitary cost for changes in the programmable load consumption profile

MILP formulation

Operational Costs

! NON-LINEAR

$$J(\cdot) = \sum_{i=1}^{m} \sum_{t=t_0}^{M-1} \left(C_i^G P_i^G(t) + C_i^B \boxed{\left| P_i^B(t) - P_i^B(t-1) \right|} + C_i^L \boxed{\left| P_i^L(t) - \widetilde{P}_i^L(t) \right|} \right)$$

Re-formulation

$h_i^B(t)$ $\quad h_i^L(t)$ $\quad$ auxiliary variables subject to:

$$\begin{cases} h_i^B(t) = \left| P_i^B(t) - P_i^B(t-1) \right| \\ h_i^L(t) = \left| P_i^L(t) - \widetilde{P}_i^L(t) \right| \end{cases}$$

$$\forall\ t = t_0, \dots, M-1$$

! NON-LINEAR

$$J(\cdot) = \sum_{i=1}^{m} \sum_{t=t_0}^{M-1} \left(C_i^G P_i^G(t) + C_i^B h_i^B(t) + C_i^L h_i^L(t) \right)$$

! LINEAR

MILP formulation

Re- Formulation

$$J(\cdot) = \sum_{i=1}^{m} \sum_{t=t_0}^{M-1} \left(C_i^G P_i^G(t) + C_i^B h_i^B(t) + C_i^L h_i^L(t)\right)$$

with $h_i^B(t)$ $h_i^L(t)$ subject to:

$$h_i^B(t) = \left|P_i^B(t) - P_i^B(t-1)\right| \qquad \left\{ \begin{aligned} P_i^B(t) - P_i^B(t-1) &\leq h_i^B(t) \\ -P_i^B(t) + P_i^B(t-1) &\leq h_i^B(t) \end{aligned} \right.$$

$$h_i^L(t) = \left|P_i^L(t) - \widetilde{P}_i^L(t)\right| \qquad \left\{ \begin{aligned} \sigma^\top \delta_i^L(t) - \tilde{P}_i^L(t) &\leq h_i^L(t) \\ -\sigma^\top \delta_i^L(t) + \tilde{P}_i^L(t) &\leq h_i^L(t) \end{aligned} \right.$$

$$\forall\ t = t_0, \ldots, M-1$$

MILP formulation

Decision Variables

$$x_i^\top = \begin{bmatrix} u_i(t_0) & h_i^B(t_0) & h_i^L(t_0) & \cdots & u_i(M-1) & h_i^B(M-1) & h_i^L(M-1) \end{bmatrix}$$

Cost Function

$$J(\cdot) = \sum_{i=1}^{m} \sum_{t=t_0}^{M-1} \left(C_i^G P_i^G(t) + C_i^B h_i^B(t) + C_i^L h_i^L(t) \right)$$

$$\begin{bmatrix} \underline{P_i^G(t) \quad P_i^B(t) \quad \delta_i^G(t) \quad \delta_i^L(t)} & h_i^B(t) & h_i^L(t) \end{bmatrix}$$

$$c_i^u = \begin{bmatrix} C_i^G & 0 & 0 & 0_{1\times J_i^L} \end{bmatrix}$$

$$u_i(t) = \begin{bmatrix} P_i^G(t) \\ P_i^B(t) \\ \delta_i^G(t) \\ \delta_i^L(t) \end{bmatrix}$$

MILP formulation

Decision Variables

$$x_i^\top = \begin{bmatrix} u_i(t_0) & h_i^B(t_0) & h_i^L(t_0) & \cdots & u_i(M-1) & h_i^B(M-1) & h_i^L(M-1) \end{bmatrix}$$

Cost Function

$$J(\cdot) = \sum_{i=1}^{m} \sum_{t=t_0}^{M-1} \left(C_i^G P_i^G(t) + C_i^B h_i^B(t) + C_i^L h_i^L(t) \right)$$

$$\begin{bmatrix} P_i^G(t) & P_i^B(t) & \delta_i^G(t) & \delta_i^L(t) & h_i^B(t) & h_i^L(t) \end{bmatrix}$$

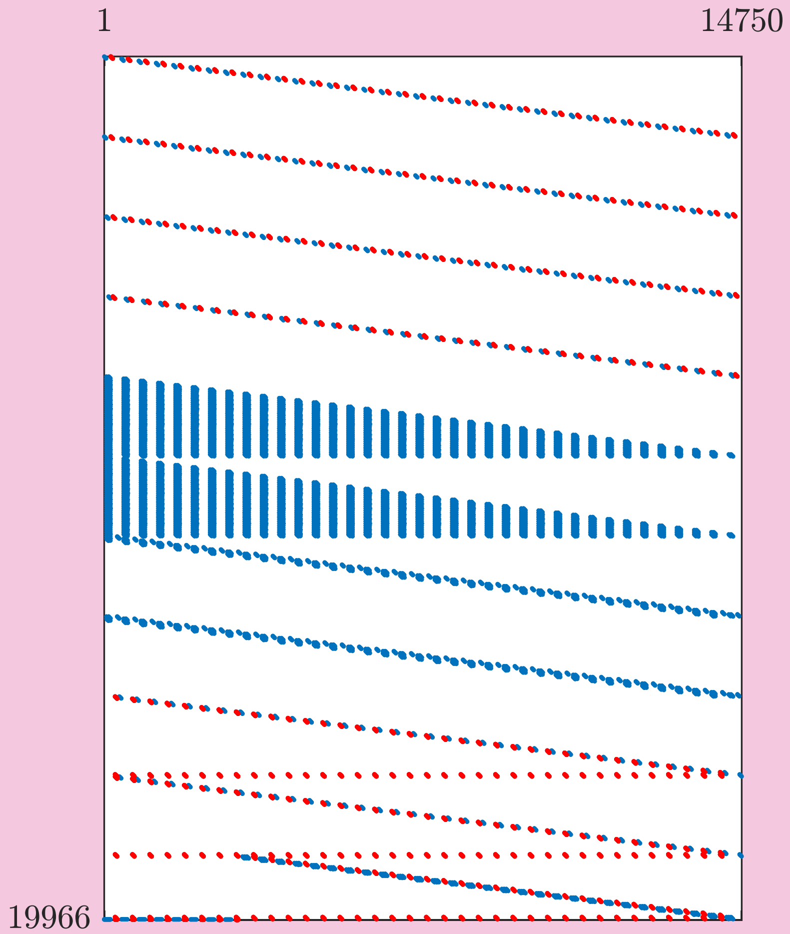
$$c_i^u = \begin{bmatrix} C_i^G & 0 & 0 & 0_{1\times J_i^L} \end{bmatrix} \qquad C_i^B \qquad C_i^L$$

$$c_i^\top = \begin{bmatrix} c_i^u & C_i^b & C_i^L & \cdots & c_i^u & C_i^b & C_i^L \end{bmatrix}$$

$$\Rightarrow \quad \boxed{J(\cdot) = \sum_{i=1}^{m} c_i^\top x_i}$$

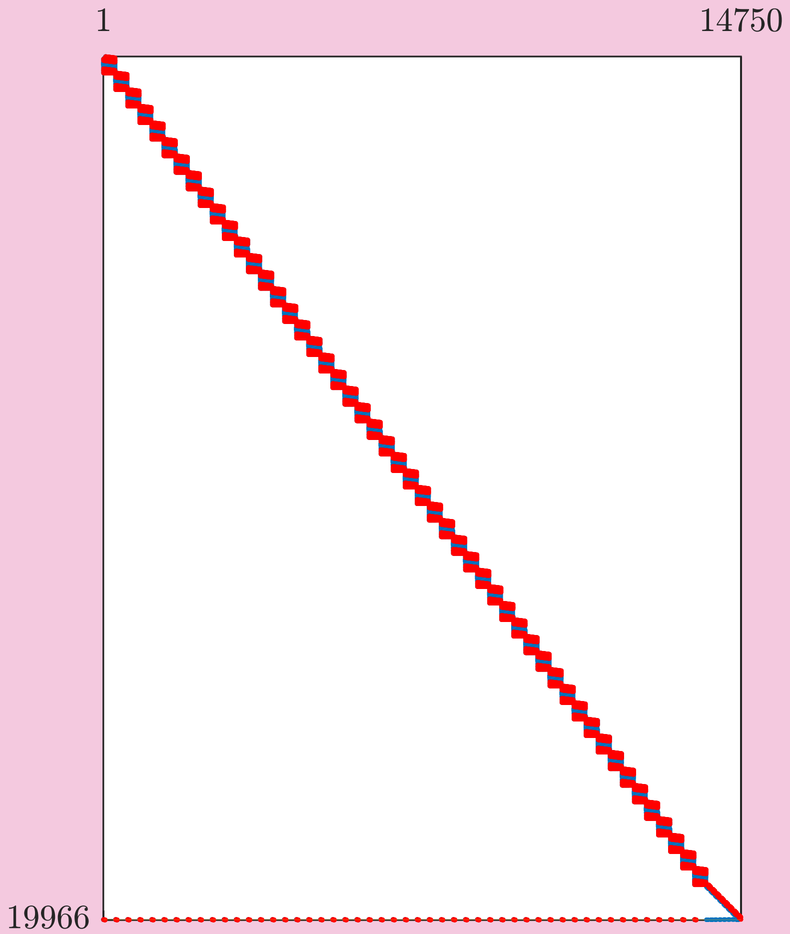
Mixed Integer
Linear Program

$$\begin{aligned} \min_{x} \quad & c^\top x \\ s.t \quad & \boxed{A} x \leq b \\ & x \in \mathbb{R}^{n_c} \times \mathbb{Z}^{n_d} \end{aligned}$$



Mixed Integer
Linear Program

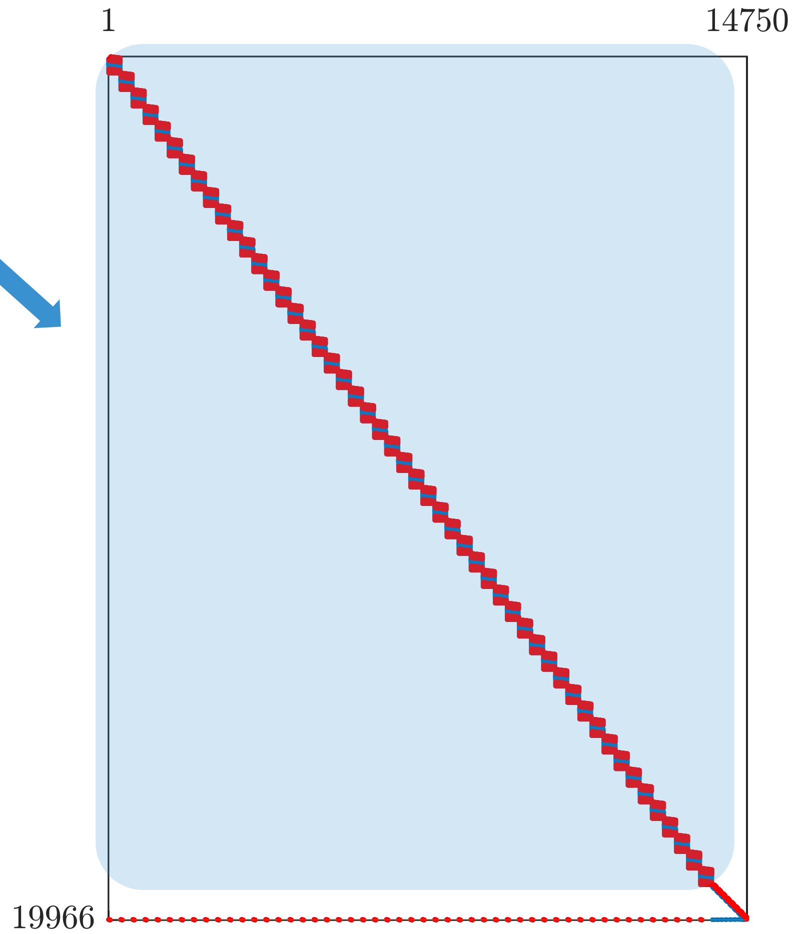
$$
\begin{aligned}
\min_{x} \quad & c^\top x \\
s.t \quad & Ax \le b \\
& x \in \mathbb{R}^{n_c} \times \mathbb{Z}^{n_d}
\end{aligned}
$$



Multi-agent constraint coupled MILP

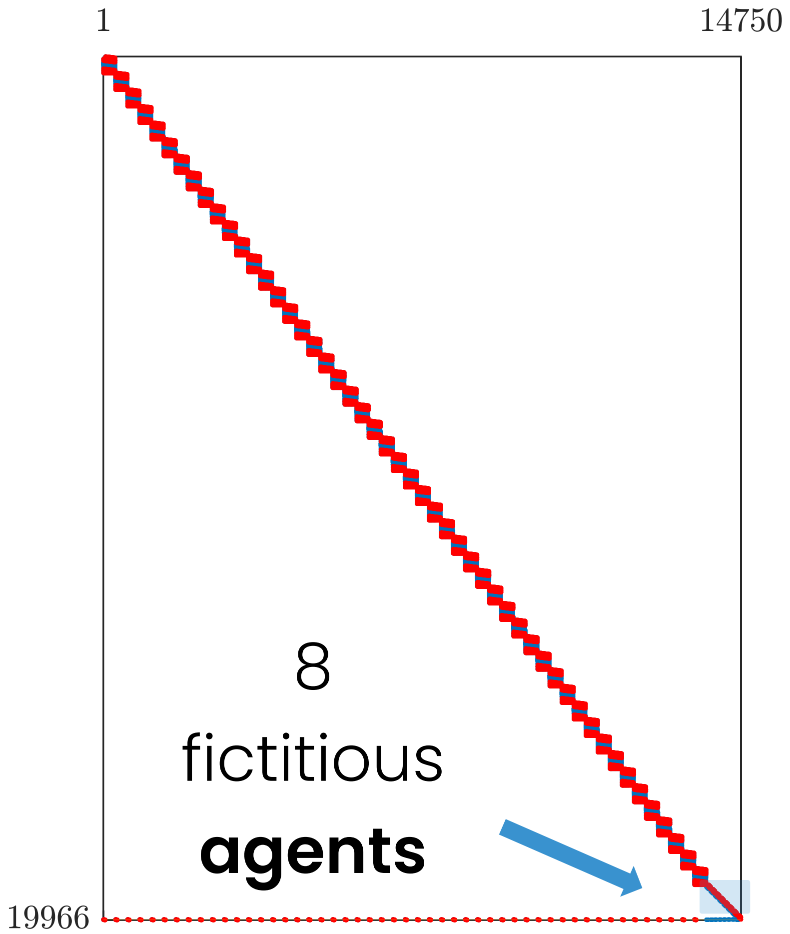
$$
\begin{aligned}
\min_{x_1,\dots,x_m} \quad & \sum_{i=1}^{m} c_i^\top x_i \\
\text{s.t.} \quad & \boxed{D_i x_i \leq d_i \quad \forall i} \\
& \sum_{i=1}^{m} E_i x_i \leq f \\
& x_i \in \mathbb{R}^{n_{c_i}} \times \mathbb{Z}^{n_{d_i}} \quad \forall i
\end{aligned}
$$

50 **prosumers**



Multi-agent constraint coupled MILP

$$\begin{aligned} \min_{x_1,\dots,x_m} \quad & \sum_{i=1}^{m} c_i^\top x_i \\ \text{s.t.} \quad & D_i x_i \le d_i \quad \forall i \\ & \sum_{i=1}^{m} E_i x_i \le f \\ & x_i \in \mathbb{R}^{n_{c_i}} \times \mathbb{Z}^{n_{d_i}} \quad \forall i \end{aligned}$$



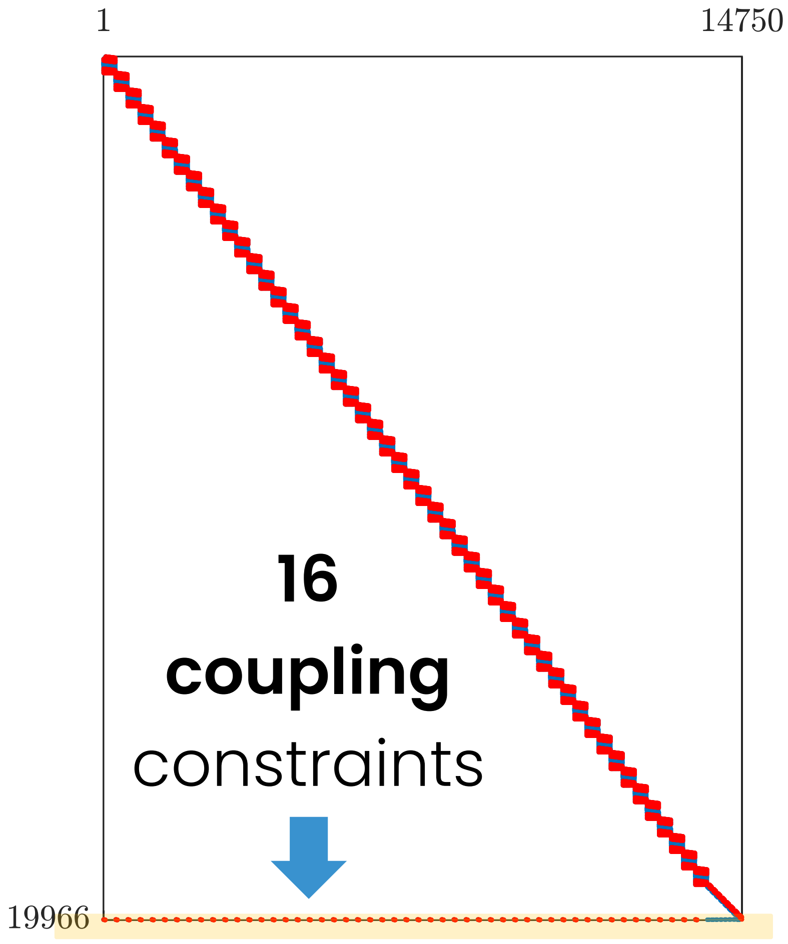
Multi-agent constraint coupled MILP

$$\min_{x_1,\dots,x_m} \quad \sum_{i=1}^{m} c_i^\top x_i$$

$$\text{s.t.} \quad D_i x_i \le d_i \quad \forall i$$

$$\sum_{i=1}^{m} E_i x_i \le f$$

$$x_i \in \mathbb{R}^{n_{c_i}} \times \mathbb{Z}^{n_{d_i}} \quad \forall i$$



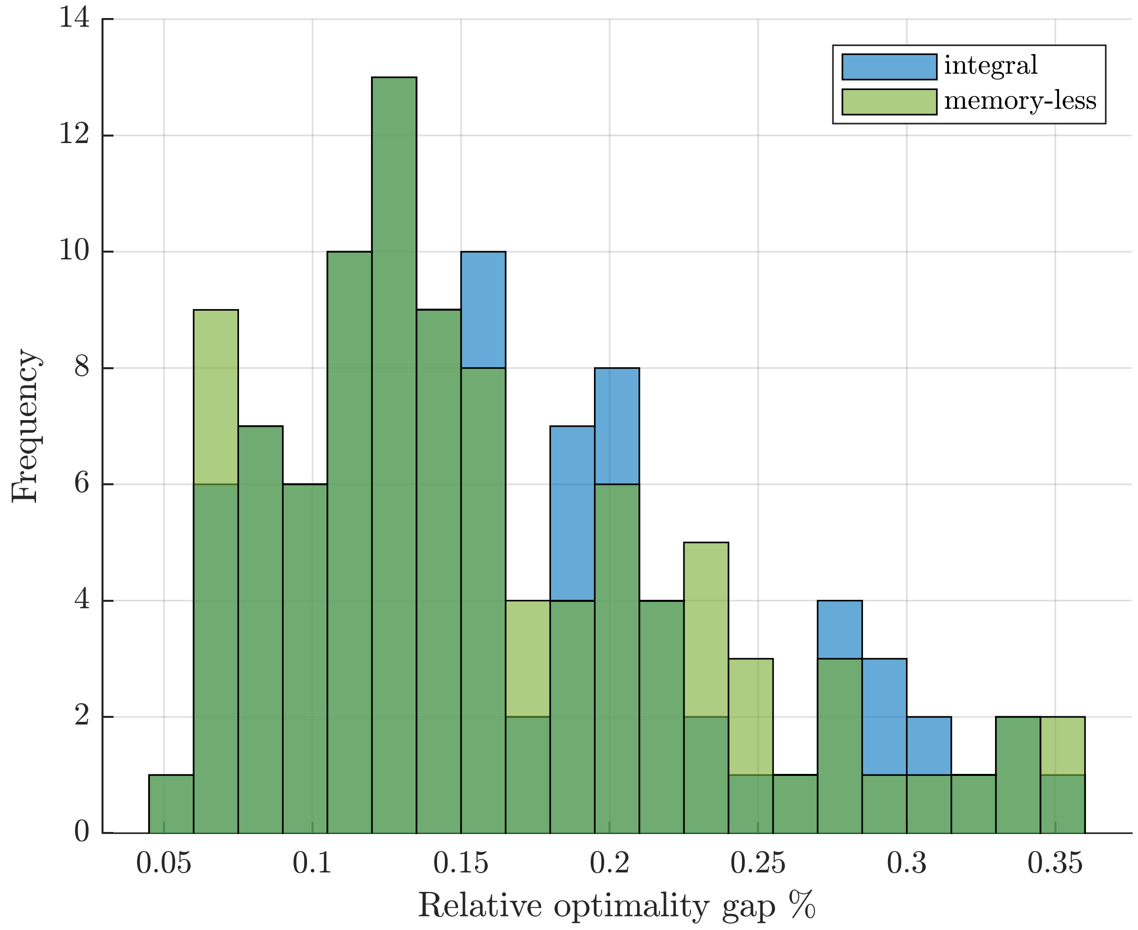
Provision of ancillary services to the power distribution grid

Comparison with **50 prosumers** over 100 different parameters sets

Quality measured based on a bound on the **distance from the optimal cost**

Provision of ancillary services to the power distribution grid

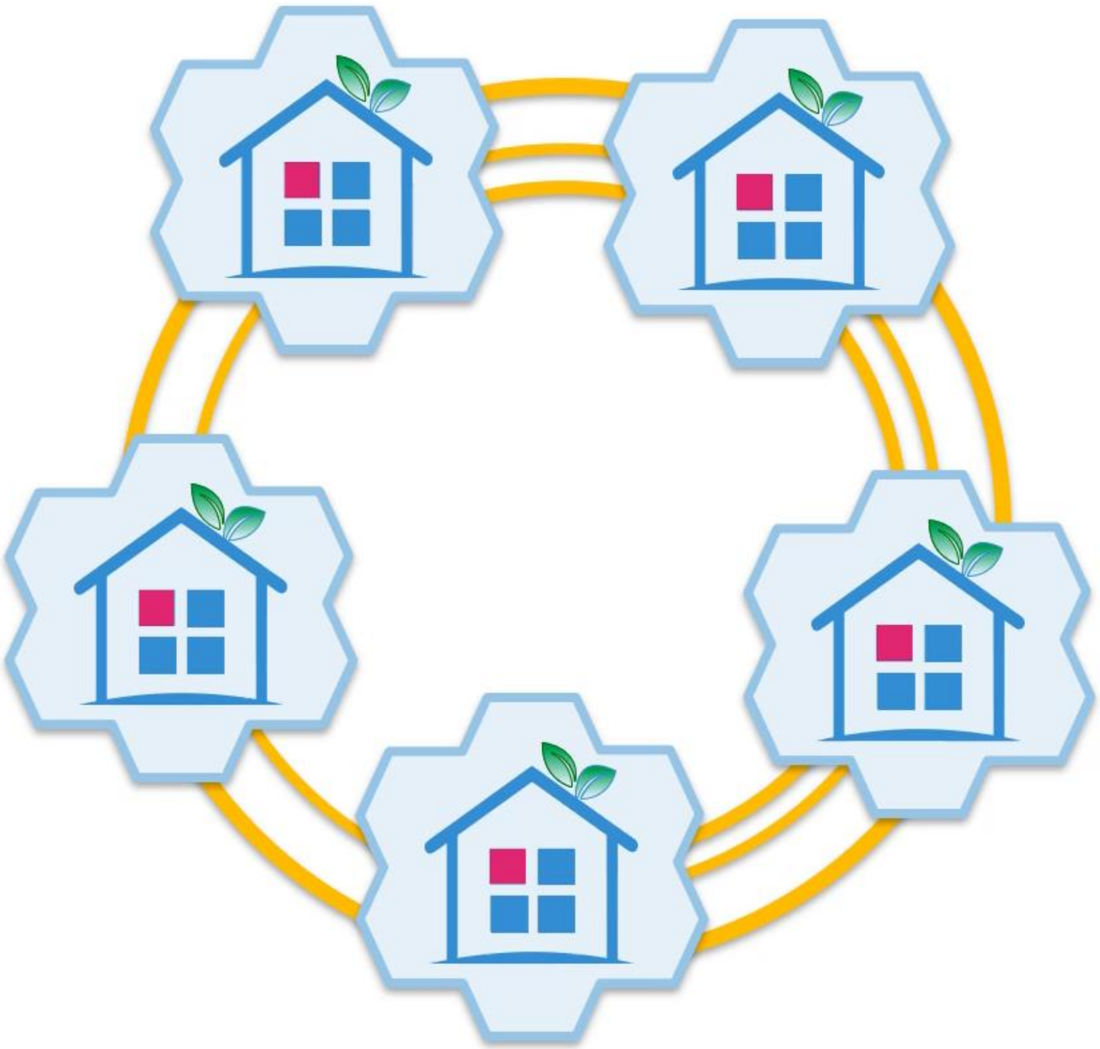
Comparison with **50 prosumers** over 100 different parameters sets



Both integral and memory-less resolution schemes achieve an **average gap** of **0.17%**

State of the art approaches could **not** compute a **feasible** solution

Provision of ancillary services to the power distribution grid



Both algorithms return **close-to-optimal** solutions as the **size** of the problem **increases**.

Using ad-hoc procedures allows to **recover** computational **tractability**

Dealing with uncertainty

Uncertain multi-agent problem

$$
\begin{aligned}
\min_{x_1,\ldots,x_m} \quad & \sum_{i=1}^{m} c_i^\top x_i \\
\text{s.t.} \quad & D_i x_i \leq d_i \quad \forall i \\
& \sum_{i=1}^{m} E_i x_i \leq f \\
& x_i \in \mathbb{R}^{n_{ci}} \times \mathbb{R}^{n_{di}} \; \forall i
\end{aligned}
$$

Uncertain multi-agent problem

$$
\begin{aligned}
\min_{x_1,\ldots,x_m} \quad & \sum_{i=1}^{m} c_i^\top x_i \\
\text{s.t.} \quad & D_i(\delta) x_i \leq d_i(\delta) \quad \forall i \\
& \sum_{i=1}^{m} E_i x_i \leq f \\
& x_i \in \mathbb{R}^{n_{ci}} \times \mathbb{R}^{n_{di}} \; \forall i
\end{aligned}
$$

Uncertain parameter

Uncertain multi-agent problem

Nominal value

$$\min_{x_1,\dots,x_m} \quad \sum_{i=1}^{m} c_i^\top x_i$$

$$\text{s.t.} \quad D_i(\bar{\delta})x_i \le d_i(\bar{\delta}) \quad \forall i$$

$$\sum_{i=1}^{m} E_i x_i \le f$$

$$x_i \in \mathbb{R}^{n_{ci}} \times \mathbb{R}^{n_{di}} \; \forall i$$

Replacing δ with some **nominal value** $\bar{\delta}$ may lead to **infeasibility**

Robust multi-agent problem

$$
\begin{aligned}
\min_{x_1,\dots,x_m} \quad & \sum_{i=1}^{m} c_i^\top x_i \\
\text{s.t.} \quad & D_i(\delta) x_i \leq d_i(\delta) \quad \forall \delta \in \Delta \ \forall i \\
& \sum_{i=1}^{m} E_i x_i \leq f \\
& x_i \in \mathbb{R}^{n_{ci}} \times \mathbb{R}^{n_{di}} \ \forall i
\end{aligned}
$$

All admissible values

Enforce constraints for **all admissible values** of $\delta \in \Delta$ leads to **overly conservative** approaches (and requires **knowledge of** Δ)

Data-driven multi-agent problem

Collected realizations

$$\begin{aligned}
\min_{x_1,\dots,x_m} \quad & \sum_{i=1}^{m} c_i^\top x_i \\
\text{s.t.} \quad & D_i(\delta)x_i \leq d_i(\delta) \quad \boxed{\forall \delta \in S_{N_i}} \; \forall i \\
& \sum_{i=1}^{m} E_i x_i \leq f \\
& x_i \in \mathbb{R}^{n_{ci}} \times \mathbb{R}^{n_{di}} \; \forall i
\end{aligned}$$

Data-based formulation using **realizations of the uncertain parameter** to characterize Δ and underlying $\mathbb{P}$.

Data-driven multi-agent problem

Collected realizations

$$
\begin{aligned}
\min_{x_1,\dots,x_m} \quad & \sum_{i=1}^{m} c_i^\top x_i \\
\text{s.t.} \quad & D_i(\delta)x_i \leq d_i(\delta) \quad \boxed{\forall \delta \in S_{N_i}} \ \forall i \\
& \sum_{i=1}^{m} E_i x_i \leq f \\
& x_i \in \mathbb{R}^{n_{ci}} \times \mathbb{R}^{n_{di}} \ \forall i
\end{aligned}
$$

?

How many data are needed for the solution to satisfy the constraints associated with not yet seen scenarios?

Existing a-priori probabilistic guarantees

[1] provides guarantees for an **optimal solution**

Finding an **optimal solution** is often **unviable**

Guarantees in [2] require a **shared data-set**

Agents may not be willing to **share sensitive info**

[1] Esfahani Peyman M. et al. **"Performance bounds for the scenario approach and an extension to a class of non-convex programs."** *IEEE Transactions on Automatic Control 60 (2014) 46–58.*

[2] Falsone Alessandro et al. **"Uncertain multi-agent MILPs: A data-driven decentralized solution with probabilistic feasibility guarantees."** *Proceedings of machine learning research (2020) 1000–1009.*

Dealing with uncertainty

Data-based guarantees

With confidence $(1-\beta)$,

$\mathbb{P}\{\delta$: a data-driven feasible solution is **infeasible** for some $i\} \leq \varepsilon$ if

$$N_i \geq \bar{N}_i\left(\frac{\varepsilon}{m}, \frac{\beta}{m}, k_i\right)$$ for **each agent** $i = 1, \dots, m$

Manieri Lucrezia et Al., " **Probabilistic feasibility in data-driven multi-agent non-convex optimization** "
Annual Reviews in Control 56 (2023)

Dealing with uncertainty

$$N_i \geq \frac{5}{\varepsilon_i}\left[2n_{ci}\log_2(4e\boxed{k_{ci}})\ln\left(\frac{40}{\varepsilon_i}\right)+\ln\left(\frac{4}{\beta_i}\right)+\ln(\boxed{k_{di}})\right]$$

number of linear inequality constraints of agent i involving continuous variables and affected by δ

number of combinations for the discrete variables of agent i.

Dealing with uncertainty

Data-based guarantees

With confidence $(1-\beta)$,

$\mathbb{P}\{\delta$: a data-driven feasible solution is **infeasible** for some $i\} \leq \boldsymbol{\varepsilon}$ if

$$N_i \geq \bar{N}_i\left(\frac{\varepsilon}{m}, \frac{\beta}{m}, k_i\right)$$ for **each agent** $i = 1, \dots, m$

The **smaller** ε**, the more samples** $\bar{N}_i$ are needed

Dealing with uncertainty

Data-based guarantees

With confidence $(1-\beta)$,

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- The **smaller ε, the more samples** $\bar{N}_i$ are needed
- **Confidence can be close to 1** since $\bar{N}_i$ grows with the logarithm of β

Dealing with uncertainty

Data-based guarantees

With confidence $(1-\beta)$,

$\mathbb{P}\{\delta$: a data-driven feasible solution is **infeasible** for some $i\} \leq \boldsymbol{\varepsilon}$ if

$$N_i \geq \bar{N}_i\left(\frac{\varepsilon}{m}, \frac{\beta}{m}, k_i\right) \quad \text{for } \textbf{each agent } i = 1, \dots, m$$

- The **smaller** ε**, the more samples** $\bar{N}_i$ are needed
- **Confidence can be close to 1** since $\bar{N}_i$ grows with the logarithm of β
- The bound **grows** with the **complexity** k_i of the **local** feasibility set

Dealing with uncertainty

Existing a-priori guarantees

[1] provides guarantees for an **optimal solution**

Guarantees in **[2]** require a **shared data-set**

Proposed a-priori guarantees

Guarantees hold for **sub-optimal solutions**

Guarantees are local and hold for **private data-sets**

[6] Esfahani Peyman M. et al. **"Performance bounds for the scenario approach and an extension to a class of non-convex programs."** *IEEE Transactions on Automatic Control 60 (2014) 46–58.*

[7] Falsone Alessandro et al. **"Uncertain multi-agent MILPs: A data-driven decentralized solution with probabilistic feasibility guarantees."** *Proceedings of machine learning research (2020) 1000–1009.*

Conclusions

Conclusions

Decomposition strategy

Disclose the **hidden** multi-agent **structure**

Decompose the problem in **smaller sub-problems**

Conclusions

Decomposition strategy

Resolution schemes

Disclose the **hidden** multi-agent **structure**

Decentralized schemes for **constraint-coupled MILPs**

Decompose the problem in **smaller sub-problems**

Less conservative and with **finite-time convergence**

Conclusions

Decomposition strategy	*Resolution schemes*	*Data-based guarantees*
Disclose the **hidden** multi-agent **structure**	**Decentralized** schemes for **constraint-coupled MILPs**	Guarantees for **sub-optimal solutions**
Decompose the problem in **smaller sub-problems**	**Less conservative** and with **finite-time convergence**	Preserve **privacy** of local **information**

Useful references

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[6] **“Performance bounds for the scenario approach and an extension to a class of non-convex programs”.**
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[7] **“Uncertain multi-agent MILPs: A data-driven decentralized solution with probabilistic feasibility guarantees”.**
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Learning for Dynamics and Control, 2020 : 1000-1009

Credit

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Credit

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NextGenerationEU

Thank you for your attention!

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