Semidefinite Programming for Domain Randomization in LQR

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Abstract

Uncertainty in model parameters presents a major challenge in control and policy design. We introduce a **semidefinite programming (SDP) framework** for domain randomization (DR) in Linear Quadratic Regulation (LQR). Our method designs a single controller to optimize the average performance and stabilizes all sampled system instances.

Introduction and Motivation

- Conventional methods, such as Min-Max control, tend to be conservative.
- DR generalizes control performance by minimizing the expected objective function over parameter distribution.
- We propose an SDP framework [1] to address this problem in LQR, which **facilitates constraint optimization**.

Problem Formulation

We consider the linear uncertain system:

$$x_{k+1} = A(\theta)x_k + B(\theta)u_k + w_k, \quad u_k = Kx_k.$$

The DR-LQR objective:

$$\begin{aligned} J_{\mathrm{DR}}(K) &= E_\theta \left[\mathrm{Tr}\left((Q + K^\top RK)\Sigma_\theta\right)\right] \\ \text{s.t.} \quad \Sigma_\theta &= (A(\theta) + B(\theta)K)\Sigma_\theta(A(\theta) + B(\theta)K)^\top + I. \end{aligned}$$

Goal is to find K minimizing $J_{\mathrm{DR}}(K)$ while ensuring stability for all samples. We estimate the expectation with M samples:

$$J_{\mathrm{DR}}(K) \approx J_{\mathrm{SA}}(K) = \frac{1}{M}\sum_{j=1}^{M} \mathrm{Tr}\left[(Q + K^\top RK)\Sigma_j\right]$$

where $\Sigma_j = (A_j + B_jK)\Sigma_j(A_j + B_jK)^\top + I$.

SDP Descent (SDPD): an iterative solution

Starting from an initial stabilizing K_0, in any iteration i find the best perturbation direction δ_i which minimizes the first-order approximation of $J_{SA}(K_i + \delta_i)$. In particular, solve:

$$\begin{aligned} \delta_i^\star = \arg\min_{\delta_i,\, \sigma_{j,i}} \quad & \frac{1}{M}\sum_{j=1}^{M} \mathrm{Tr}[(Q + K_i^\top RK_i)\sigma_{j,i} + 2K_i^\top R\delta_i\Sigma_{j,i}] \\ \text{s.t.} \quad & \sigma_{j,i} \succeq A_{j,i}\sigma_{j,i}A_{j,i}^\top + A_{j,i}\Sigma_{j,i}(B_j\delta_i)^\top + B_j\delta_i\Sigma_{j,i}A_{j,i}^\top, \\ & \forall j \in \{1, \ldots, M\}, \ \|\delta_i\| \le \eta_i, \end{aligned}$$

and update by $K_{i+1} = K_i + \delta_i^\star$ if $J_{SA}(K_i + \delta_i^\star) \le J_{SA}(K_i)$, otherwise, decrease the step size η_i and repeat.

Proposition 1 (Stability) *A sufficient condition to remain stable for each sample j at iteration i is*

$$\|\delta_i\| < \mu_{j,i} := \frac{\|A_{j,i}\|}{\|B_j\|}\left[\sqrt{1 + \frac{1}{\|A_{j,i}\|^2\,\|\Sigma_{j,i}\|}} - 1\right]. \tag{1}$$

Proposition 2 (Perturbation sensitivity) *as $\|\delta_i\| \to 0$,*

$$\|\Sigma_{j,i}(K_i + \delta_i) - \Sigma_{j,i}(K_i)\| = \mathcal{O}(\|\delta_i\|).$$

Jointly Stabilizing Initial Controller (JSIC)

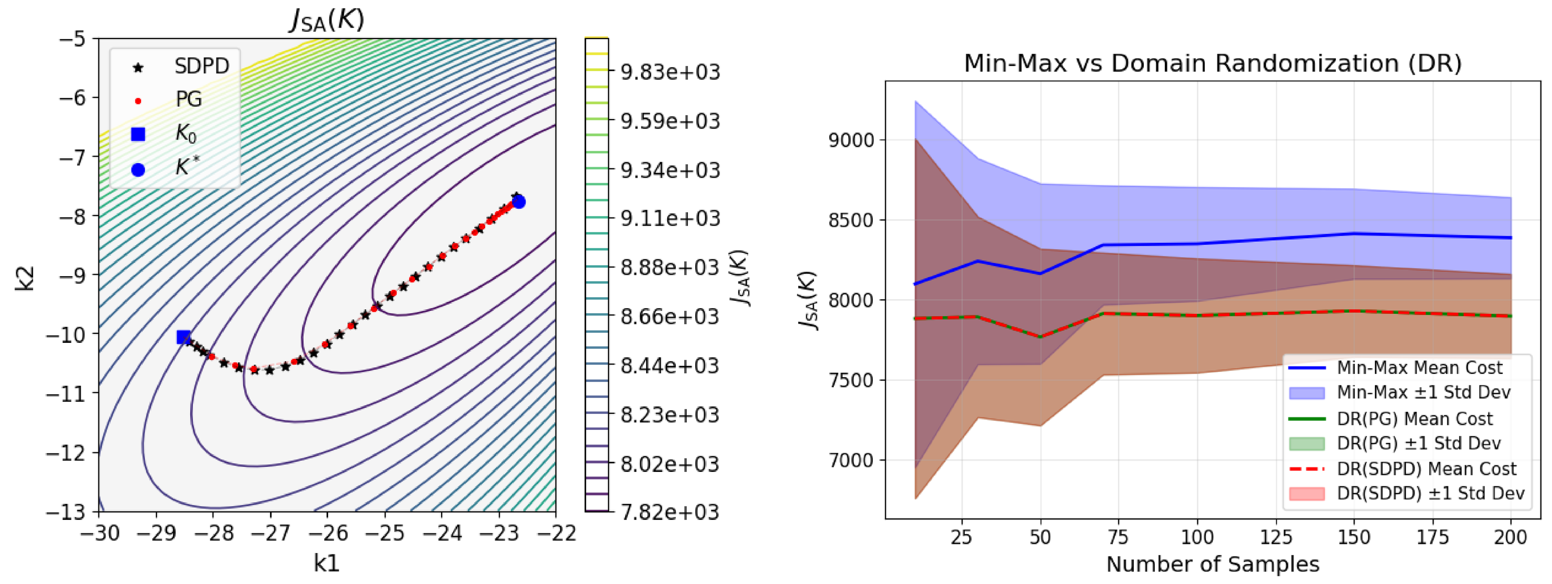
A stabilizing feedback gain K_0 exists if and only if the optimum of the following problem is less than one; *i.e.* $\alpha < 1$

$$\begin{aligned} \min_{K_0,\, P_j,\, \alpha_j} \quad & \alpha \\ \text{s.t.} \quad & (A_j + B_jK_0)P_j(A_j + B_jK_0)^\top \prec \alpha_j P_j, \\ & \alpha > \alpha_j, \quad P_j \succ 0, \quad \forall j \in \{1, \ldots, M\}. \end{aligned} \tag{2}$$

This problem can be solved using a similar approach to SDPD.

Simulation Result

- Discretized and linearized inverted pendulum is used.
- Fig. (a) The trajectory of convergence for the SDPD and PG [2] algorithms. Fig. (b) DR approach vs Min-Max for H_2 controller.



(a) (b)

Contribution to SEDDIT

DR in LQR yields optimal controllers that adapt well to real-world variability, managing energy consumption. In particular, our method:

- can handle variations in the parameters of the dynamics enabling optimal energy usage,
- reduces inefficiencies due to overdesign for worst-case scenarios by cutting unnecessary energy consumption.

Acknowledgements

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Reference

[1] A. Pasdar et al., "Semidefinite Programming for Domain Randomization in LQR", Submitted to European Control Conference (ECC) 2026.

[2] T. Fujinami et al., "Policy Gradient for LQR with Domain Randomization", arXiv:2503.24371.

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